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Abstract

We present strong evidence that financial bubbles can be identified ex-ante and that a sharp price increase, when suitably qualified by a bubble indicator called LPPLS confidence, does on average predict unusually low returns going forward. For this, we use a methodology called log-periodic power law singularity (LPPLS) combined with the event study method applied to industry sectors in China over 2005–2020 and the US over 2009–2020. We identify a new class of apparent bubble regimes corresponding to the convergence to a stable price level, which can be disentangled using LPPLS-based indicators from standard bubbles followed by crashes.

Keywords: Bubbles, Log-periodic power law singularity, Confidence indicator, Bubble crash, drawdown

JEL: C20, C40, C53, G01, G17

*These violent delights have violent ends. And in their triumph die, like fire and powder.
Which, as they kiss, consume.*

————— *William Shakespeare*

1. Introduction

Eugene F. Fama is known for rejecting the existence of bubbles based on the lack of sufficient *ex-ante* evidence and of a systematic identification methodology. In his Nobel lecture, Fama (2014) defines a bubble as an “irrational strong price increase that implies a predictable strong decline.” Based on their post-mortem analysis of industry statistics, Greenwood, Shleifer, and You (2019) also claim that “a sharp price increase of an industry portfolio does not, on average, predict unusually low returns going forward.” We object to this conclusion by presenting a systematic methodology for identifying bubbles at the industry level. Specifically, we use the log-periodic power law singularity (LPPLS) methodology developed by the Sornette research group over more than two decades. We claim that the LPPLS-based methodology in a straightforward operational implementation can allow for an *ex-ante* and causal (in real-time) identification of bubbles, especially in assets experiencing a strong *super-exponential* price increase¹ (Sornette and Johansen, 2002; Sornette, 2003). Our evidence confirms that a *super-exponential* price change—characterizing a strong, irrational price movement—is *unsustainable*, and hence it causes a break in the pre-existing price dynamics.

The models of standard neoclassical economics fail to incorporate the nonlinear dynamics and complex non-equilibrium and non-stationary properties of financial markets. These properties transcend the explanatory abilities of neoclassical financial theories. The European Central Bank’s (ECB) Governor Jean-Claude Trichet points out

¹ We define a bubble as a “super-exponential” price increase, which is followed by a sudden collapse. “Super-exponential” indicates that the growth rate of the price (or average return) grows itself. Recall that a constant growth rate means that the price grows exponentially, which is the standard regime of financial assets owing to proportional growth and the law of compounding interests. When the growth rate grows itself, the price accelerates hyperbolically, which is unsustainable.

this limitation in relation to the Great Financial Crisis, in 2010², in the following quote: “As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools.” In this context, we elaborate on the insights of Sornette and his co-workers, who have built on the existence of several positive feedback mechanisms in financial markets that make them intrinsically unstable (Minsky, 1972, 1992). Specifically, this instability takes the form of long-lived phases of super-exponential price acceleration (Johansen and Sornette, 2010; Ardila-Alvarez, Forro, and Sornette, 2021), followed by regime changes manifested through sharp corrections or volatile plateaus.

These alternating phases of transient price acceleration and corrections call for modifying the standard mean reversion model of market dynamics to reflect the long-lived transient, unstable regimes self-correcting in abrupt market “*ruptures*.” The modified model should capture that the market reverts only in the long-term and in a strong nonlinear style. Figure 1 vividly illustrates such nonlinear regime shifts in the Hang Seng Index from 1970 to 1997. At that time, the Hong Kong market presented a textbook example of free capital flows driven by market forces, which contributed to a succession of stock market and property bubbles. In the figure, the straight line followed by abrupt ruptures represents large excursions away from the average annual growth. Specifically, the figure shows several periods where the log-price trajectory exhibits an upward curvature, indicating a transient super-exponential growth until the onset of a sharp drawdown. The figure outlines the eight largest transient episodes, though there were other episodes at smaller scales. The logarithmic scale in the Figure 1 should clearly indicate that, during a bubble episode, the price shoots up by 50% to 1000% within a span of a few months to a few years.

² Opening address by Jean-Claude Trichet, President of the ECB, at the ECB Central Banking Conference Frankfurt, 18 November 2010. (<https://www.ecb.europa.eu/press/key/date/2010/html/sp101118.en.html>)

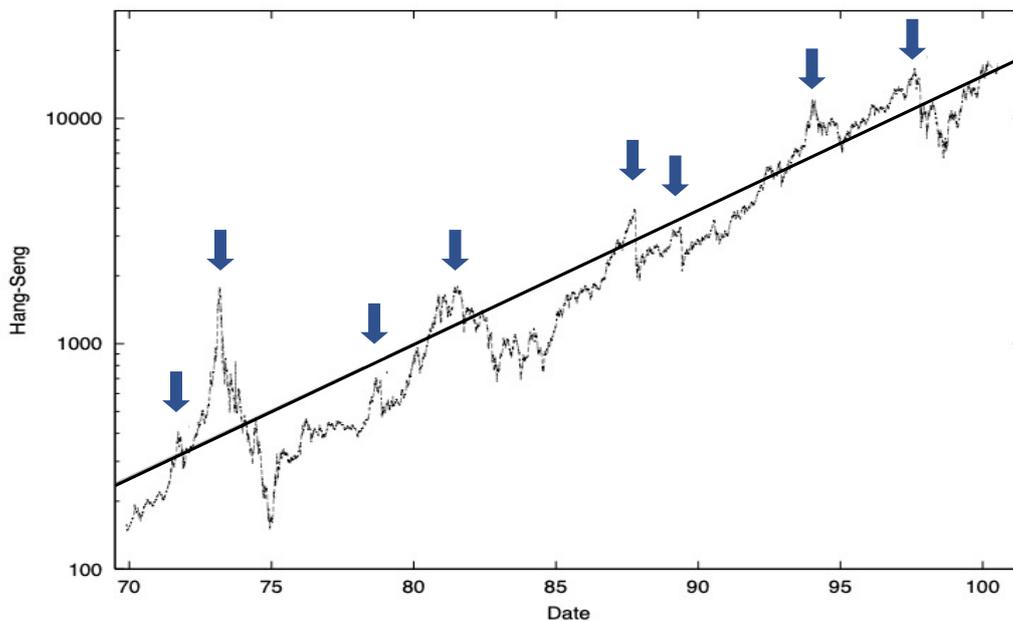


Figure 1: The Hang-Seng composite index of the Hong Kong stock market from November 1969 to September 1999. The culmination of the bubbles followed by strong corrections or crashes corresponding to a drawdown larger than 15% in less than three weeks is indicated by arrows at October 1971, February 1973, September 1978, October 1980, October 1987, April 1989, January 1994, and October 1997. The straight line shows an average exponential growth rate of $\approx 13.6\%$ per year. (Adapted from Sornette and Johansen (2001)).

In this context, it must be understood that fund managers and professional investors cannot avoid bubbles. Investors should not only hold large positions in the right stocks and buy them at bargain prices but also hope to cash out their positions close to the peak, and a failure to do so may spoil their efforts³. However, the key problem is how to determine the right time—neither too early nor too late.

Consider the dot-com bubble to further illustrate the conundrums that investors face. At that time, the legendary investor and founder of the Tiger Fund, Julian Robertson, refused to follow the herd owing to its irrational crowd behavior and

³ Investors who fail to time the market peak before the crash might face the curse of Sisyphus. According to Greek mythology, Sisyphus tries to push a boulder to the mountaintop. However, every time he reaches the mountaintop, the stone rolls down and he starts pushing the boulder uphill again.

consequent market overvaluation. In 1999, the Tiger Fund was dissolved because its value stocks could not keep pace with the technology stocks. Stanley Druckenmiller, who at that time was the manager of the Quantum Fund, also detected market overvaluation. He thus exploited the market, overplayed his hand, and suffered a massive loss in 2000 when the crash occurred before he could unwind his positions. Although both Julian Robertson and Stanley Druckenmiller were disciplined and intelligent investors and they clearly predicted a bubble (as confirmed later by the crash), they both suffered losses for different reasons and hold unhappy memories of the event. While both detected the irrational behavior, the former avoided the technology stocks, while the latter made an ill-timed short bet^{4,5}.

The fund managers who leave the market too early earn negligible profits and are shadowed by peers who enjoy the appreciation of the bubble, while the fund managers who leave the market too late lose part of, all, or more than the capital gains accumulated during the appreciation. Managers who make the highest profit are not deceived by misrepresentations and lies, ride the trend, and step off before it is discredited (getting out of the market before the crash). We suggest that this objective can be achieved by using a suitable measure of unsustainable price dynamics as explained below.

This study has thus two objectives. First, we present a LPPLS model using advanced confidence indicators to detect bubbles (positive and negative) at the industry-group level in both China and the US market. Second, we use the event study method to record price behaviors systematically, before and after the rise of LPPLS-based confidence indicators. We also categorize bubbles according to the magnitude of

⁴Legendary value investor Charles de Vaulx's refusal to join the *crowd* and the performance pressure led to his suicide in 2021 (<https://www.barrons.com/articles/legendary-value-manager-charles-de-vaulx-found-dead-51619564407>)

⁵For example, the failure of Archegos Capital in 2021 can be attributed to its overplay in the herding trend—a combination of leverage and momentum-chasing led to the margin call. (<https://www.moneycontrol.com/news/business/personal-finance/vital-lessons-for-investors-from-the-archegos-saga-6713791.html>)

the LPPLS confidence indicators. To the best of our knowledge, this is the first study to identify bubbles at the industry-group level in both China and the US and to use event study to examine the predictability of the LPPLS model.

We present three major findings. First, bubbles can be detected *ex-ante* and causally by using the LPPLS confidence index, unlike the claim by Fama (2014) and Greenwood et al. (2019) that bubbles cannot be identified in real time. Our finding proves that prices contain sufficient information to achieve bubble detection capabilities.⁶ Second, bubbles frequently appear at the industry-group level in the Chinese and US markets. Particularly, the Chinese market has more collective bubbles (signaling an impending price decline) that simultaneously develop in different industries, despite their different business cycles. In contrast, the bubbles in the US market exhibit a more uniform distribution (signaling a large impending price decline) over a period, suggesting more decoupling. Third, positive bubbles and negative bubbles do not have symmetric shapes in the Chinese and US markets. Specifically, a larger positive LPPLS confidence indicator suggests overreaction and a subsequent strong price decline (bubble crash or large drawdown), while a smaller positive LPPLS confidence indicator implies underreaction and a subsequent plateauing of the price. For negative bubbles⁷, a negative LPPLS confidence indicator with a large amplitude quantifying an accelerated price decline signals higher future volatilities. The upward price rebound that sometimes follows a price decline is more apparent in the US than that in the Chinese market.

The study most similar to ours is titled *Bubbles for Fama* by Greenwood et al. (2019), which uses historical industry-level data for both US and other countries to test whether significant stock run-ups will lead to crashes; however, their paper differs from ours in its event selection methodology, i.e., they use fixed thresholds for the industry price increases (e.g., a 100% price run-up during two consecutive years) to filter their

⁶ We are the first to apply the LPPLS Confidence Indicator to the US and Chinese industry-level datasets. The empirical evidence reported in this study in particular on Chinese markets can be considered an "out-of-sample" test of the LPPLS Confidence Indicator.

⁷ A negative bubble is characterized by a fast-accelerating declining price, which is symmetric to a positive bubble via the symmetry $\ln [p(t)] \rightarrow -\ln [p(t)]$.

events and to analyze what happens afterward, while we use the LPPLS Confidence Indicator to identify the event date, observing the market behavior after the event date. Based on the general criterion of a significant deviation from the standard exponential growth, this method can be considered agnostic with respect to the mispricing level qualifying a bubble, and hence more flexible to adapt to different bubble price structures. Our study also differs from Greenwood et al. (2019) in that the latter uses monthly returns data, while we use daily return data. For transient bubble episodes, monthly data may be too coarse-grained; for example, a bubble lasting 12 or 24 months will have only 12 or 24 data points. Our daily price resolution is more adapted to capture transient abnormal super-exponential price dynamics, given that our model uses a dedicated calibration to account for large daily residuals. Based on their methodology, Greenwood et al. (2018) conclude that considerable price increases do not lead to market crashes, but when the stock price goes up a lot, the probability of a substantial crash increases. In contrast, we come to the conclusion that strong super-exponential price increases, detected by higher LPPLS Confidence Indicators, will be followed by substantial market declines (crashes). The opposing conclusions can be attributed to the fact that our methodology is better adapted to bracket the peaks of the bubbles. A fixed threshold, as in Greenwood et al. (2018), such as a 100% price run-up during two consecutive years, may lead to the selection of a time that does not capture the full course of the bubble. The time at which the 100% price run-up threshold is reached may be when the bubble is still running its course upward and not when it bursts. The averaging over many such configurations taken at random times (corresponding to the arbitrary 100% price run-up) in the lifetime of the bubbles may lead to the misleading conclusion that there is no subsequent price decline following a bubble. In other words, by averaging over cases when the bubble is still growing strongly and when the price is in its drawdown phase after the bubble has burst, it is logical that Greenwood et al. (2018) conclude that the average behavior does not exhibit substantial market declines.

The conclusion of our study is similar to that of Hong and Stein (1999) in their classification of the post-event price performance. Our results also provide a conceptual framework to reconcile the short-term underreaction and long-term overreaction patterns in financial markets. Our paper is also similar in spirit to Kinlaw et al. (2018), as we both aim to time the market at the industry level. Kinlaw et al. (2018) used the

asset centrality measure based on a Principal Component Analysis of the covariance matrix of the returns of a set of sectors and argued that large asset centrality is indicative of crowded trading, which is often associated with the formation of bubbles. In contrast, we use the LPPLS model that builds on signatures associated with positive feedbacks to identify the end of bubbles. We find that our LPPLS-based approach outperforms Kinlaw et al.'s method.

Two other papers related to our work are Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). Barroso and Santa-Clara (2015) show that a run of a momentum portfolio enjoying a remarkably good performance is occasionally followed by a large crash, which makes the strategy risky. They show that the tail risk can be managed by controlling the realized variance of daily returns. Daniel and Moskowitz (2016) use momentum premium and volatility to generate portfolios with dynamic asset weights that have a better Sharpe ratio and larger alpha than Barroso and Santa-Clara's static momentum portfolio. Daniel and Moskowitz (2016) focus on portfolio risk management, while we adopt a straightforward approach of directly predicting which kind of price increase (acceleration) is likely to lead to a crash.

The remainder of this paper is organized as follows. Section 2 presents a short review of the relevant literature. Section 3 explains the theory underpinning and methodologies implementing the LPPLS model and event study. Section 4 presents the data, and Section 5 discusses the empirical findings. Section 6 concludes the study.

2. Literature review

The efficient market hypothesis (EMH) implies the unpredictability of future returns and the absence of bubbles that can be exploited for arbitrage opportunity. Fama (1965) reckons that, although behavioral/boundedly rational traders exist in financial markets, sophisticated arbitrageurs correct any exploitable potential mispricing induced by these behavioral traders. However, a large body of research exhibit evidence that market inefficiency (limits of arbitrage such as costs, risks, and restrictions) and bounded rationality (cognitive limitations, information imperfection, and constraints) may promote the existence of transient bubbles. Several empirical findings provide evidence of financial bubbles, and hence may be in contrast with the EMH. In this

regard, it must be noted that, owing to momentum trading, trend chasing, and the likes, a large number of *behavioral* agents are subjected to animal spirits, fads and fashions, and overconfidence, which might lead to positive feedback loops. In this context, psychological and behavioral elements of stock price determination may make the stock return predictable, at least in pockets of predictability associated with the end of the bubble regimes as we discuss below.

2.1 Background and historical attempts to define bubbles

Bubbles refer to the significant, sustained mispricing of financial or real assets. According to the basic asset pricing theory, the asset value should be equal to the discounted future fundamental income flow, otherwise, arbitrage opportunities will arise. However, the difficulty in accurately predicting the future fundamental income flow *ex-ante* poses a real-world problem. Abstracting from the economic issues, any trivial change in the assumptions of the valuation model can generate a large range of current fundamental values (Black, 1986). Thus, the use of such ambiguous characterization of the fundamental value always muddles the evidence for bubbles.

Some financial economists deny the existence of bubbles because they believe in efficient markets (the price reflects all publicly available information) (Fama 1965, 1970; Samuelson, 1965, 1973). As per their theory, rational speculators stabilize the price, and sophisticated investors do not allow for the emergence of a bubble (Fama, 1965; Friedman, 1953). In efficient markets, there are no free lunch. In contrast, Shiller (1984) suggests that the argument behind the EMH is invalid because it ignores the agents' psychology and the interactions between agents at the origin of price fluctuations. He also claims that market values cannot be justified based on future dividend flow, thus supporting the existence of bubbles (Shiller, 1986; 2015). Keynes (1936) also suggests the possible existence of speculative bubbles. Scheinkman and Xiong (2003) show that, although the aggregate beliefs of investors may be unbiased, large fluctuations in heterogeneous beliefs can lead to the formation of significant bubbles. According to behavior finance theory, the limits to arbitrage and the presence of irrational investors may also lead to the formation of bubbles.

Concerning its definition, the exact definition of bubbles varies in the financial literature (Brunnermeier, 2012). Kindleberger (1978) defines a bubble as an upward

price movement over an extended range that then implodes. Santoni and Dwyer (1990) describe a bubble as a regime that appears when the market does not follow a random walk. Stiglitz (1990) defines a bubble as a situation followed by marked price declines, which occur without any apparent new information. Using the Q-theory of investment, McGrattan and Prescott (2001) define bubbles as periods when the value of a set of assets exceeds the sum of the values of individual assets. Siegel (2003) defines a bubble as any time the realized asset return over a given future period is more than two standard deviations from its expected return.

Concerning modeling the bubbles, Schatz and Sornette (2020) propose a mathematical framework accommodating discrete and continuous-time bubble models featuring market inefficiencies. This framework provides a solid theoretical background to embed feasible asset price processes during financial speculation and frenzy. Through this framework, Schatz and Sornette (2020) demonstrate that the rational expectation bubble models suffer by design from the paradox that a rational market should not allow for mispricing, both in discrete and continuous cases. They also show that this problem is not solved within the finite time *strict local martingale* approach to bubbles (Jarrow, Kchia, and Protter, 2011; Protter, 2013).

In line with Schatz and Sornette (2020), we classify the bubble literature into two groups. The type-I group considers bubbles as the premium of a future crash risk leading to exorbitant (inefficient) stock price development. A typical representative member of this group is the rational expectation bubble model. This group is further sub-divided into the symmetric and asymmetric information bubble groups. The type-II group argues that bubbles represent a temporary departure from market efficiency. These prices return to efficient levels (e.g., through efficient crashes) after the correction of inefficiency occurring during a drawdown. Theories underpinning the bubbles of the type-II group include heterogeneous beliefs, behavior finance and complex system theories. As per the *theory of heterogeneous beliefs*, bubbles result from prior heterogeneous beliefs in the asset fundamental value. The *behavior finance theory* holds that bubbles arise owing to limits to arbitrage or positive feedback caused by noise or not fully rational investors. *Complex system theory* argues that bubbles emerge owing to the imitation and collective herding behavior of heterogeneous agents. Specifically, the repetitive nonlinear interaction between agents may create positive

feedback loops leading to speculative bubbles. The following sections elaborately explain these approaches.

2.2 Rational Expectation Bubble

2.2.1 Rational Expectation Bubble under symmetric information

2.2.1.1 Rational bubble according to Blanchard and Watson

Rational bubbles emerge owing to investors' self-fulfilling expectations about future price trajectories (Blanchard, 1979). The agents fully understand the fundamental asset value but are willing to pay more than the underlying value. This can occur if agents' expectations of future price growth are high enough to compensate for the risk they are willing to take. In other words, the expected return is higher than the required rate of return, and the investors can sell the asset, on average, at a profit in the future.

In this regard, motivated by non-fundamental factors (e.g., sentiment and over-optimism, etc.), Blanchard and Watson (1982) propose a model in which the asset price can be decomposed into a fundamental and non-fundamental factor,

$$P_t = V_t + B_t \quad (1)$$

where P_t is the asset market price, V_t indicates the asset fundamental value, and B_t represents the bubble component at time t .

Blanchard and Watson (1982) consider that the bubble, expected to grow at a long-term growth rate of \bar{r} , persists with probability π and bursts with a probability of $(1 - \pi)$. The bubble has a growth rate $(1 + \bar{r}) / \pi$ to ensure a fair risk-neutral valuation condition. Thus, during the time that the bubble persists, it has to grow faster than the historical average return.

Lux and Sornette (2002) have investigated the statistical properties of rational expectation bubbles and found that the distribution of returns belongs to the class of so-called heavy-tailed distribution so that the tails of the distribution of returns follow approximately a power law distribution with a tail exponent smaller than 1 (owing to the rational expectation conditions). This contradicts the stylized facts of financial data that the tail exponent is in the range 2 to 4. The rational exponential bubble models of this class thus fail at a very elementary level.

2.2.1.2 *Intrinsic bubble according to Froot and Obstfeld*

Froot and Obstfeld (1991) proposed a specific rational bubble model under symmetric information, also known as the intrinsic bubble model. A prerequisite of this model is that the bubble component is determined by the dividends (a proxy of fundamentals) in a non-linear deterministic approach. This model suggests that the bubble emerges because of overreactions to the stochastic dividends. The price dynamics reads:

$$\hat{P}_t = \hat{V}_t + B(D_t) \quad (2)$$

$$B(D_t) = cD_t^\gamma \quad (3)$$

where \hat{P}_t is the asset market price, \hat{V}_t is the fundamental value, B denotes the bubble component, and D_t is the dividend at time t; $c > 0$ and $\gamma > 1$ are parameters.

The bubble component is deterministically related to the dividends, as the exponential growth of the bubble follows the growth rate of the dividends. If the fundamentals remain the same (i.e., the dividends remain constant), the bubble component will remain the same. However, if the fundamental shows an up-side shift (i.e., the dividends show an up-side shift) or investors expect the fundamental to be up-side shifting, the bubble component will cause an accelerated surge in the asset market price.

While the intrinsic bubble model is also a self-fulfilling bubble, it is driven by fundamental expectations. In contrast, it is the non-fundamental factors that drive the rational bubble of Blanchard and Watson.

2.2.2 *Rational Expectation Bubble under Asymmetric Information*

Under the assumption of symmetric information, a bubble is commonly known by all rational investors. However, in the case of asymmetric information, everybody is aware of the existence of a bubble, but they do not know whether other investors are aware of its existence. The asymmetric information theory assumes that investors share a common prior belief distribution, but possess different information. Under this assumption, the price has the following two roles: (i) price reflects the aggregate information, and (ii) price itself is the informative signal, as it induces others to partially reveal their aggregate information.

It must be noted that the lack of higher-order knowledge makes it possible for a finite bubble to emerge. Allen et al. (1993) proposed the *contrapositives* of the no-trade theorem, highlighting the following necessary conditions for the existence of bubbles. First, the price cannot be fully revealed, as the investors remain asymmetrically informed after exposure to prices and net trade information. Second, investors have limited short-selling ability, which makes the bubbles persist. Third, the market structure, which assumes that the initial allocation is interim Pareto efficient, is not common knowledge.

Tirole (1982) showed that, if market information is common knowledge, then there is no gain from trading. Any bid or offer to initiate a transaction will reveal the bidder's private knowledge and place them in an unfavorable situation. In other words, trading requires that investors believe that they gain something from the trade. For instance, fund managers who invest for their clients buy assets even during the bubble stage. This convinces the clients that their managers have superior information that drives their superior performance. An inability to trade will reveal their lack of superior private information that can lead to profit. Given this, bad fund managers tend to *churn bubbles* by taking on overvalued assets, despite knowing that they might be the *last in line* (Allen and Gorton, 1993).

2.3 Inefficient Market Bubble

Fama has been a staunch upholder of the hypothesis that markets are efficient. He also believes that an observed excess return is not the result of investing in an inefficient market but that it is a risk premium associated with other unknown risk factors. However, a large body of literature seems to contradict Fama's view. These works suggest that a systematic error (i.e., mispricing) owing to *inefficient* or not fully rational agents can lead to excess returns.

2.3.1 Heterogeneous Belief Bubble

Investors may disagree about the fundamental value of the asset or may be constrained by short-selling. In such a scenario, investors hold overpriced assets if they can resell them to someone less informed or, in Kindleberger's (1978) terms, to a

greater fool. The prices of these assets are determined at equilibrium to reflect the heterogeneous beliefs. Beliefs differ, given that investors have different prior belief distributions because of psychological biases. For example, heterogeneous beliefs and short-selling restrictions force the pessimists out of the market as they fail to counterbalance the high asset prices determined by the optimists. Miller (1977) shows that the optimists inflate the new equilibrium to a higher level. Simsek (2013) proposes that the disagreement on beliefs constrain optimists' ability to borrow from pessimists. Intuitively, pessimists are more inclined to fund optimists when the disagreement is about the upside than the downside state.

Lintner (1969), Chen, Hong, and Stein (2002), and Kurz (1996) show that heterogeneous beliefs with short-sale constraints restrictions can lead to overpricing. Scheinkman and Xiong (2003) show that a high trading volume and price volatility are features of heterogeneous-belief bubbles. Ofek and Richardson (2003) and Cochrane (2003) link this argument to the dot.com bubble of the late 1990s. In this context, Sohn and Sornette (2020) propose an extension of the class of rational expectations bubbles to the more general rational beliefs setting of Kurz (1994), in which a heterogeneous population of agents can hold more than one *rational* expectation. This can lead to the emergence of correlated beliefs allowing for the convergence of rational but diverse beliefs. These beliefs may not cancel each other in aggregate, and this can make them an object of rational speculation. Thus, diverse but correlated beliefs can account for speculative bubbles, without the need for irrational agents or limits to arbitrage.

2.3.2 Behavior Finance Theory

EMH reasons that bubbles do not exist because the sophisticated arbitragers (rational investors) eliminate mispricing opportunities introduced by irrational investors. However, behavior finance theory holds that markets are inefficient because (i) there is a limit to arbitrage, and (ii) investors do not always process information correctly. Costs and risks also deter sophisticated investors and professional arbitragers from eliminating the arbitrage opportunities.

2.3.2.1 Short selling restrictions

Lee et al. (1999) state that price and value form a co-integrated system and converge in the long-term owing to arbitraging. However, as a result of exogenous forces (e.g., arbitrage costs) and constraints (e.g., short-selling constraints), prices can depart from their fundamental value. For example, Almazan et al. (2004) indicate that roughly 70% of the mutual funds explicitly claim that they have short-selling restrictions. Koski and Pontiff (1999) show that 79% of the equity mutual funds do not access derivatives, suggesting that they cannot use synthetic ways to take short positions. The failure of short-sellers to correct overpricing might also be caused by sentiment-driven traders who exacerbate overpricing (De Long, Shleifer, Summers, and Waldmann, 1990a; Shleifer and Vishny, 1997).

2.3.2.2 Costs of Arbitrage

Grossman and Stiglitz (1980) show that the market cannot be perfectly informationally efficient because information acquisition is costly. If the market were efficient, then investors spending resources to obtain knowledge would receive no compensation and would stop such investments, thus removing the forces pushing the market toward its efficiency frontier. Shleifer and Vishny (1997) believe that arbitrage costs can be divided into the following three subclasses: trading, holding, and information costs. Trading costs include brokerage fees, price slippage, bid-ask spreads, and other costs related to building or closing the positions. Holding costs comprise costs for sustaining the positions of various durations and implementing short-selling. Information costs are associated with accessing, analyzing, and monitoring information.

2.3.2.3 Fundamental Risk

Fundamental risk refers to the arbitrageurs being wrong about the subsequent fundamental news of the targeted companies and thus taking wrong arbitrage positions, which leads to losses. For instance, rational investors might short a bubble, but the subsequent positive shift in fundamentals might unwind the arbitrage strategies, if the initial overpricing is justified by the new fundamentals.

Theoretically, the fundamental risk can be hedged by taking a reverse position of a closely related asset. For example, the arbitrageur has a long position on shares A and is aware of their downside risk. In theory, the best strategy would be to short shares B, which is a perfect substitute for shares A. However, there are no perfect substitute securities of shares A, and shares B themselves might be mispriced. Thus, it is challenging to eliminate fundamental risk (Barberis and Thaler, 2003)

2.3.2.4 Noise Trade Risk

Kyle (1985) and Black (1986) refer to noise traders as irrational investors. They also point out that the EMH does not account for noise trading, which could explain daily excess trading and unreasonable price movements. According to the noise trader model, market prices and the company's fundamental value deviate from each other over a relatively long term (De Long et al., 1990). In addition, noise traders' activities pose a risk to all market participants, which impacts the valuation anchor of the market by disturbing the cost of capital. Shleifer and Vishny (1997) show that arbitrageurs have short investment horizons because they often manage the money of investors with different time preferences. The unpredictable behavior of noise traders creates a risk in the assets, deterring rational arbitrageurs from aggressively betting against them. In addition, the performance of the arbitrageurs is usually judged based on their short-term returns because any temporary losses might trigger fund outflows, which might unwind their position when the mispricing is the largest.

2.3.2.5 Synchronization Risk

Abreu and Brunnermeier (2002, 2003) argue that bubbles cannot be offset unless arbitrageurs coordinate their actions, given that individual investors do not play a significant role in controlling an overpriced market. Thus, lack of coordination is another reason for persistent mispricing. Rational arbitrageurs know that market bubbles will collapse as soon as an adequate number of rational traders sell the stocks. However, the dispersion of rational arbitrageurs' opinions regarding the timing of the bubble inception delays their arbitrage orders (i.e., sell-off). This is attributed to the fact that, if arbitrageurs attack the bubble too early, they will forgo profits owing to the

subsequent run-up caused by momentum traders. Given this, arbitrageurs uncertain about the beginning of the bubble might find it optimal to ride rather than attack the still growing bubble. Demos and Sornette (2017) further show that determining the start of a bubble is easier than determining the end of a bubble. Technically, the maximum likelihood estimation of the time of the bubble inception is much more rigid than the estimation of the end time of the bubble, which is more inaccurate in a technical sense. Therefore, Demos and Sornette (2017, 2019) argue that bubbles persist owing to arbitrageurs' difficulty to synchronize their belief about the end rather than about the beginning of the bubble.

Concerning this synchronization risk, based on the Securities and Exchange Commission's equity position data from 1998 to 2000, Brunnermeier and Nagel (2004) investigate the holdings of hedge funds (i.e., funds of well-known managers such as Paul Tudor and George Soros). They find that hedge fund portfolios were weighted more to the technology segments during the dot.com bubble. These sophisticated investors started reducing their holdings on a stock-by-stock basis before the collapse of some stocks. However, they switched to other technology stocks that still had a rising trend. The excess returns the hedge funds enjoyed in the technology segment support the existence of synchronization risk and disprove the claims of Friedman (1953) and Fama (1965) that rational speculators will stabilize the price and sophisticated investors will not allow the emergence of bubbles.

2.3.2.6 Positive Feedback Caused by Noise Traders

Behavior finance theory maintains that investors have inherent psychological biases, such as conservatism and overconfidence, and are subject to the disposition effect and the representativeness bias. These psychological biases can lead to an overreaction or underreaction behavior, both of which lead to positive feedback dynamics at the origin of momentum effects.

Jegadeesh and Titman (1993) develop a pioneer study of the momentum effect arising from the positive feedback of trading strategies that buy or sell assets with superior and inferior past performance. Ardila-Alvarez et al. (2021) introduce the *acceleration* effect (and factor) defined as the variation in the momentum over a suitable time frame. They show that this positive feedback strategy even overperforms

the momentum factor to a large extent at the time of bubbles and crashes. Fama and French (1993, 1996) claim that most return reversals and other anomalies can be explained mainly by the three-factor model, except for the momentum effect. Fama and French (2015) extend their three-factor model to a five-factor model in order to address the parts of stocks' average return patterns unexplained by the three-factor model. However, the five-factor model still fails to explain the momentum factor (Fama and French, 2018).

According to Edwards (1968), investors tend to underweight new information when updating their information set. De Bondt and Thaler (1985) find overreaction in the stock market by showing that investors are influenced by waves of optimism and pessimism that cause prices to deviate systematically from their fundamental values; however, in the long-term, prices exhibit mean reversion. Lehmann (1990) and Jegadeesh (1990) document short-term reversal effects of one-week and one-month, respectively. Cross-sectional return predictability exists in many markets (Rouwenhorst, 1998) and appears between and within industries (Maskowitz and Grinblatt, 1999; Hameed et al., 2010).

Barberis et al. (1998) explain that the conservatism bias might lead investors to underreact to information, generating momentum profits. The conservatism bias suggests that prices slowly adjust to new information and that returns have no further predictability once the price reflects this information. Grinblatt and Han (2005) propose that the disposition effect leads to underreaction, which means that loss-averse investors anchoring on past prices tend to hold on to their past losers and sell their past winners. De Long et al. (1990) describe the delayed overreaction bias by showing that positive feedback trading strategies, where one buys past winners and sells past losers, cause prices to deviate from their fundamental value. Andreassen and Kraus (1990) empirically demonstrate that investors tend to chase price trends corresponding to positive feedback trading based on extrapolative expectations.

Based on the representative heuristic proposed by Tversky and Kahneman (1974), Barberis et al. (1998) argue that investors tend to mistakenly conclude that firms realizing consistent extraordinary earnings growth are more likely to continue to offer similar extraordinary increases in the future. This representativeness causes prices to overshoot in the short term and reverse to the fundamental value over the long term.

Daniel et al. (1998) hypothesize that informed traders have self-attribution bias, which means that traders attribute the performance of ex-post winners to their stock selection skills while attributing a bad performance to simple bad luck. As a result of this cognitive bias, these investors overestimate the precision of their signals for asset valuation. However, their overconfidence in their signals pushes up the past winners beyond their fundamental value. The delayed overreaction in this model results in momentum profits and reversal behaviors.

Hong and Stein (1999) propose a unified underreaction and overreaction model. They consider two groups of investors—news-watching and momentum-trading investors—assuming that private information diffuses gradually across the news-watching investor population. Accordingly, the new information obtained by news-watching investors is transmitted with delays. Thus, it is progressively reflected in the price, which leads to short-term underreaction and the momentum effect. Conversely, the momentum traders, who trade based on limited price history and who arbitrage away any underreaction left behind by the news-watchers, push the past winners above their fundamental values. This behavior eventually leads to long-term price reversals.

2.3.3 Complex System Theory of Bubbles and crashes

2.3.3.1 Social Imitation, Collective Herding, Bifurcation, and Phase Transitions

Imitation and herding might be the most visible imprint of human beings' behaviors in our social affairs. Psychologists and neuroscientists have studied imitation as one of the humans' most evolved cognitive processes; it requires sophisticated brain processing abilities and a very developed cortex. According to the social brain hypothesis of R. Dunbar (1998), imitation and other social skills co-evolved with the brains of mammals, in the form of evolutionarily advantageous traits, to strengthen group cohesion. Dunbar (1998) shows that humans have the largest cortex-to-brain ratio, the most sophisticated social and imitative aptitudes, and the largest social group sizes among mammals. Moreover, it is optimal to imitate others when we lack sufficient time, energy, and information based only on private information and limited processing ability (Roehner and Sornette, 2000). Hirshleifer (2015) emphasizes the effect of social

interactions and contagion behavior in the propagation of individual cognitive biases, calling for more work beyond behavioral finance.

The main message of complex system theory is that a system (comprising several heterogeneous interacting agents with repetitive interactions) tends to self-organize its internal structure, leading to surprising *emergent*⁸ out-of-equilibrium properties. For instance, the tendency for humans to imitate others can result in herding and crowd effects at the macroscopic level. Cooperative herding and social imitation within the group can form positive feedback, reinforcing interaction among the agents and leading to virtuous or vicious circles. A central property of a complex system is that the repetitive non-linear interactions among its constituents can lead to large-scale collective behavior with a rich structure (Goldenfeld and Kadanoff, 1999; Sornette, 1999, 2002, 2003).

A typical complex system such as the financial market has a group of competing agents subjected to a myriad of influences. With exogenous news and endogenous interactions, the market can develop self-organized and self-reinforcing extreme behaviors. Bubbles and crashes will also *emerge* through a self-organization process (Kaizoji et al., 2015; Westphal and Sornette, 2020). Mathematicians call this extreme behavior *bifurcation* or *catastrophe* (Thom, 1989). Statistical physicists call it *phase transition* (Stanley, 1987). As per the *bifurcation theory*, a trivial change in circumstance, interaction strength, or heterogeneity (e.g., avalanche, earthquake, or financial crash) could lead to a sudden and dramatic shift in behavior at the macro-level.

2.3.3.2 *Nonlinear Positive feedback bubble*

Roehner and Sornette (2000) propose that the fundamental ingredient of bubbles is rooted in the repetitive actions of interactive nonlinear influences leading to large-scale

⁸ Emergence refers to the formation of rich collective behaviors in a complex system that cannot be predicted from the rules of interactions, at the microscopic level of elementary agent or components. Emergence is the opposite of the representative agent approach, given that macro behavior is expected to be fundamentally different from the individual agents' micro-behaviors.

correlations and eventually catastrophic events. During a bubble, the progressively increasing build-up of cooperativity and active interactions between investors in the market can form the herding behavior. The self-reinforcing positive feedback mechanism propels the market price to accelerate in a *super-exponential* way, which eventually reaches a critical point beyond which a crash can occur in the form of an abnormal drawdown. Johansen and Sornette (2002; 2010) define a crash as a *drawdown outlier*, i.e. an abnormally large cumulative loss over consecutive days of negative returns, which can be interspersed among small positive returns, occurring after the critical point.

According to complex system theory, the specific origin of the prices' collapse is not the most important issue—the market crashes because the market has entered into an unstable phase. Trivial disturbances or news can reveal the underlying instability of the market and trigger the crash. Over-leveraged financial institutions, complex derivatives, the expansion of credit, immature technology and innovations, and delusional mood are contributors to the positive feedback mechanisms at the origin of financial market instabilities. In this view, the market crashes because the system has matured toward an instability and has reached an unsustainable critical point. Several mathematical stochastic processes have been proposed to capture the positive feedback dynamics leading to transient super-exponential price dynamics ending at critical points, also known as *finite-time singularities* (Ide and Sornette, 2002; Sornette and Andersen, 2002; Andersen and Sornette, 2004; Lin and Sornette, 2013; Lin et al., 2014; 2019). It must be noted that super-exponential bubbles exist in equity and real estate markets as well as commodities and cryptocurrency markets (Gerlach et al., 2019; Wheatley et al., 2019). While fundamental valuation tools can be used to recognize bubbles in the equity and real estate markets, this intrinsic value is more difficult to define in the commodities and cryptocurrency markets.

3. Methodology

The present work builds on the LPPLS (log-periodic power law singularity) framework. It is based on the combination of (i) positive feedback mechanisms caused by factors such as imitation and herding (Sornette 1999, 2017) and (ii) discrete scale

invariance (Sornette, 1998) associated with an approximate hierarchical structure of human groups (Zhou et al., 2005) and/or generated dynamically via the interplay between fundamentalists and chartists (Ide and Sornette, 2002). The following section explains how to use the LPPLS framework as an advanced tool to detect a speculative bubble (i.e., a super-exponential price increase followed by a crash).

3.1 The log-periodic power law singularity (LPPLS) Model

In a nutshell, the LPPLS model describes a positive bubble regime by expressing the expected log-price (Sornette, Johansen, and Bouchaud, 1996; Feigenbaum and Freund, 1996) as

$$E[\ln p(t)] \approx A + B(t_c - t)^m \{1 + C \cos [\omega \ln (t_c - t) + \theta]\} \quad (4)$$

where $A = \ln [p(t_c)] > 0$ and $B < 0$ quantifies the amplitude of the price acceleration whose shape is controlled by the exponent $0 < m < 1$, C is the magnitude of the oscillations around the power-law singular growth, ω is the angular log-frequency of the oscillations before the critical time t_c denoting the end of the bubble embodying the hierarchical discrete scale invariance of the accelerating large-scale volatility structure. θ is a phase parameter in $(0, 2\pi)$ that corresponds to an encoded time unit. The term $B(t_c - t)^m$ with $0 < m < 1$ and $B < 0$ describes the super-exponential growth of the price up to the critical time t_c (Ardila-Alvarez et al., 2021), which we propose to be characteristic of bubbles. The last cosine term accounts for a long-term volatility structure also accelerating up to t_c . It is derived from the existence of hierarchical social structures (Zhou et al., 2005) and/or the interplay between nonlinear momentum and nonlinear value investing in the presence of inertia in the trading decision-making process (Ide and Sornette, 2002). The case $B > 0$ corresponds to a negative bubble—a price that accelerates downward.

The LPPLS model was first formalized with the rational expectation bubble framework by Johansen, Ledoit and Sornette (1999) and Johansen, Sornette, and Ledoit (2000). In this framework, the price acceleration is treated as the remuneration for the exposure to an accelerated crash hazard rate. This is known as the Johansen-Ledoit-Sornette (JLS) model (for pedagogical reviews, see Geraskin and Fantazzini, 2013; Jhun, Palacios, and Weatherall, 2018). For the derivation of the LPPLS model, we also refer to Johansen et al. (1999), Johansen et al. (2000), Sornette and Johansen, (2001),

and Sornette and Cauwels (2015a). For a description of its efficient calibration, we refer to Filimonov and Sornette (2013).

The LPPLS model was first developed for studying the precursory behaviors before finite-time singularities, which pervade the solutions of (non-stochastic and stochastic) nonlinear ordinary and partial differential equations describing many systems in nature (Sornette, 1998, 2006). Examples include acoustic emissions before the rupture of engineering structures (Anifrani, Le Floch, Sornette, and Souillard, 1995) or seismic and chemical precursors of earthquakes (Sornette and Sammis, 1995; Johansen et al., 1996).

The original LPPLS model has been extended in the following studies. It was extended by constructing higher-order LPPLS ‘Landau’ versions and generalized Weierstrass-type LPPLS models (Gluzman and Sornette, 2002; Zhou and Sornette, 2003a). It has been generalized to second-order and higher harmonics (Sornette and Johansen, 1997; Sornette and Zhou, 2002). The augmented LPPLS model has been used for financial bubble modeling with macro-economic factors (Zhou and Sornette, 2006a). Studies have also derived a relationship between the LPPLS parameters from the condition that the crash hazard rate $h(t)$ must remain positive by definition (Bothmer and Meister, 2003). The model has been supplemented with the fundamental value of the stock and with the idea of an *efficient crash* (Yan, Woodard, and Sornette, 2014; Kreuser and Sornette, 2019). It has also been extended to include negative bubbles (Yan, Woodard, and Sornette, 2010). Studies have introduced the LPPLS-autoregressive (AR)(1) and LPPLS-generalized autoregressive conditional heteroskedasticity (GARCH)(1) models to reflect a mean-reverting volatility process with a stochastic conditional return (Gazola, Fernandes, Pizzinga, and Riera, 2008; Liberatore, 2010; Lin, Ren, and Sornette, 2014). Studies have recommended the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root test to verify the Ornstein-Uhlenbeck property of the LPPLS fitting residuals (Geraskin and Fantazzini, 2013).

An improvement in the LPPLS model with quantile regression has led to the introduction of LPPLS confidence and trust indicators known to provide robust alarm signals (Sornette and Cauwels, 2015; Zhang, Zhang, and Sornette, 2016). Another study has improved LPPLS by presenting rigorous likelihood methods generating interval estimates of the parameters and, in particular, of the critical time denoting the end of

the bubble (Filimonov, Demos, and Sornette, 2017) and by adjusting for the sloppiness of parameters (Bree et al., 2010). Seyrich and Sornette (2016) have provided a plausible micro-founded model for the power law finite time singular form of the crash hazard rate in the JLS model of rational expectation bubbles. We also note extensions by using the Levenberg–Marquardt algorithm (LMA) algorithm (Liberatore, 2011), likelihood inference approach (Filimonov, Demos, and Sornette, 2017), and an improved genetic algorithm gyration method (Dai, Zhang, Tarzia, and Ahn, 2018). These extensions aim at improving the calibration accuracy of the LPPLS model. As already mentioned, for explosive semi-martingales, Schatz and Sornette (2020) introduce a mathematical framework transcending the rational expectation bubble framework used in the JLS model, thus allowing for many extensions.

The LPPLS model has been used in the ex-ante diagnosis and postmortem analysis of bubbles and crashes in the stock markets as well as in bond, commodity, real estate, and cryptocurrency markets, among others. This reflects the proposed universality of the super-exponential speculative bubble in the following markets:

- **The US stock market:** Standard & Poor (S&P) 500 index (Sornette and Zhou, 2002, 2006; Zhou and Sornette, 2003a, 2003b, 2006a; Drozd, Grummer, Ruf, and Speth, 2003; Zhang et al., 2016; Gerlach, Zhao, and Sornette, 2020; Shu and Zhu, 2021), the Dow Jones index (Vandewalle, 1998; Bolonek-Lason and Kosinski, 2011; Gustavsson, Leven, and Sjogren, 2016), and Nasdaq (Johansen and Sornette, 2000a), among others.
- **The global stock market:** global stock market indexes (Johansen and Sornette, 2001; Drozd, Kwapien, Oswiecimka, and Speth, 2008), the Japanese stock market (Johansen and Sornette, 1999, 2000b; Lynch and Mestel, 2017), the Korean stock market (Ko, Song, and Chang, 2018), the German stock market (Kurz-Kim, 2012; Bartolozzi, Drozd, Leinweber, and Speth, 2005; Wosnitza and Leker, 2014a), the Brazilian stock market (Cajueiro, Tabak, and Wernecka, 2009), the Polish stock market (Gnacinski and Makowiec, 2004), the Romanian stock market (Pele, Mazurencu-Marinescu, and Nijkamp, 2013), and the Chinese stock market

(Bastiaensen, Cauwels, Sornette, Woodard, and Zhou, 2009; Jiang et al., 2010; Yan et al., 2012; Sornette et al., 2015; Chong, 2017; Shu and Zhu, 2020), among others.

- **Commodities:** Precious metals (Drozd et al., 2008; Liberatore, 2010; Akaev, Fomin, Tsirel, Korotayev, 2011a, 2011b; Geraskin and Fantazzini, 2013), and the oil bubble (Sornette, Woodard, and Zhou, 2009; Cheng, T. Fan, D. Fan, and Li, 2018), among others.
- **Property market:** Real estate in the United Kingdom (Fry, 2009; 2014; Bianchetti, Galli, Ricci, Salvatori, and Scaringi, 2016), the United States (Zhou and Sornette, 2006b, 2008; Brauers, Thomas, and Zietz, 2014), the Hong Kong and Seoul property markets (Xiao, 2010), and Switzerland (Ardila-Alvarez et al., 2013; 2017; 2018), among others.
- **Bond market:** Corporate bond yield (Clark, 2004), government bond CDS spread (Wosnitza and Denz, 2013), and financial institutions' CDS spread (Wosnitza and Leker, 2014b; Wosnitza and Sornette, 2015), among others.
- **Other applications:** Election prediction (Fry and Burke, 2020), flash crash (Matsushita and Silva, 2011), and bitcoin (Gerlach, Demos, and Sornette, 2019; Wheatley, Sornette, Huber, Reppen, and Gantner, 2019), among others.

Moreover, in August 2008, Sornette and collaborators created the Financial Crisis Observatory⁹ (FCO) at ETH Zurich. This scientific platform aims to test and quantify,

⁹ <https://er.ethz.ch/financial-crisis-observatory.html>: The FCO has been monitoring approximately 25,000 assets worldwide, including indices, stocks, bonds, commodities, currencies, and derivatives. It has constructed daily updates of several bubble indicators based on the analyses of price time series using the LPPLS model presented above. As part of the research conducted within the FCO, motivated by the fact that back-testing is subjected to several possible biases, in November 2009, the financial bubble experiment (FBE) was initiated within the FCO. The FBE was based on an innovative framework to perform secure, verifiable ex-ante forecasts on financial crises

rigorously and systematically and on a large scale, the hypothesis that financial markets exhibit a degree of inefficiency and potential for predictability, especially, during the start of the bubble regimes.

3.2 LPPLS Confidence Indicator

The LPPLS confidence indicator was introduced in the methodology of the FCO in the early 2010s and was presented to the academic community in (Sornette and Cauwels, 2015). The LPPLS confidence indicator is defined as the fraction of the fitting windows satisfying pre-defined conditions at a given time of analysis (present time). These conditions are derived from the cumulative empirical experience obtained by the previous calibration of many financial bubbles, as summarized by Zhang et al. (2016) for instance. We also perform additional qualifying tests to judge whether the calibrations are acceptable; these tests include the unit-root test of the residuals, the Lomb log-periodic tests (Sornette and Zhou, 2002), and other criteria (Sornette, 2017). A large value of the LPPLS confidence indicator suggests that the LPPLS model accurately depicts the present regime, thereby qualifying the existence of an ongoing bubble. This diagnostic is usually associated with the existence of a super-exponential price increase in the analyzed empirical time series. Conversely, a vanishing or small value of the LPPLS confidence indicator means that no time window or a few time windows can be satisfactorily fitted by the LPPLS equation, suggesting the absence of a bubble and the presence of a more normal regime.

As previously mentioned, we define the upward- and downward-accelerating price increases as positive and negative bubbles, respectively. At a given present time denoted t_2 , we calibrate a given log-price time series by the LPPLS model (Filimonov and Sornette, 2013) over a set of N time windows $[t_1, t_2]$ obtained by varying t_1 over

using a creative digital fingerprint system to ensure the authenticity of forecasts released 6 months later for verification. Forecasts were revealed only after the predicted event, with the original date in which they produced these same results being publicly and digitally authenticated (Sornette et al., 2009, 2010a, 2010b; Woodard, Sornette, and Fedorovsky, 2010). Additionally, the FCO has been providing monthly reports since February 2014 of the “bubbly” state of the major financial assets worldwide.

a certain range. We calculate the LPPLS confidence indicators over the set of time windows such that t_2-t_1 spans from 30 to 250 trading days in steps of 1 trading day, corresponding to $N=221$ time windows. We add the condition that fits are acceptable only if $t_c - t_2$ lies between 0 and $t_2 - t_1$, that is, the critical time is not too far into the future. If the LPPLS confidence indicator is non-zero with $B < 0$, it will represent a positive bubble signal and we will report a positive value for the confidence indicator. If it is non-zero with $B > 0$, it will represent a negative bubble signal and we will report a negative value for the confidence indicator (See **Appendix A** for more information on the LPPLS confidence indicator.)

3.3 Market Model for Event Study

The statistical market model we use for the event study measures the return of any given security relative to the return of the market portfolio (Ball and Brown, 1968; McWilliam and Siegel, 1997). The linear specification of the model is complemented by the assumption of joint normality for the asset returns. The market model for security i is

$$R_{it} = \alpha_i + \beta_i * R_{mt} + \varepsilon_{it}, E(\varepsilon_{it}) = 0 \text{ and } \text{var}(\varepsilon_{it}) = \sigma_{\varepsilon_i}^2 \quad (5)$$

where R_{it} and R_{mt} are, respectively, the return of security i and of the market portfolio at period t , and ε_{it} is the zero-mean residual term. α_i, β_i , and $\sigma_{\varepsilon_i}^2$ are the period- t parameters of the market model for the given security i . From expression (5), the abnormal sample return $\hat{A}R_{it}$ is defined by

$$\hat{A}R_{it} = \hat{R}_{it} - \hat{\alpha}_i - \hat{\beta}_i * R_{mt} \quad (6)$$

The cross-sectional average abnormal return ($A\hat{A}R$) over the N stocks at time t is:

$$A\hat{A}R_t = \frac{1}{N} \sum_{i=1}^N \hat{A}R_{it} \quad (7)$$

The cumulative average abnormal return ($CA\hat{A}R$) is the mean cumulative return over the entire time window of all events:

$$CA\hat{A}R_{(t_1, t_2)} = \sum_{t=t_1}^{t_2} A\hat{A}R_{it} \quad (8)$$

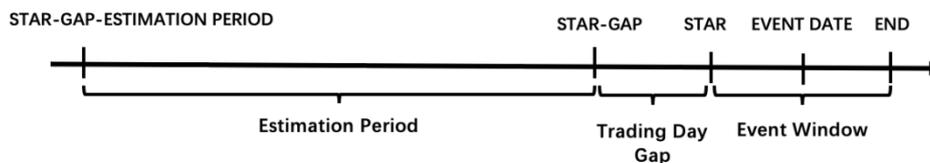


Figure 2. Timeline for an event study

We adopt the event study method (MacKinlay, 1997). Figure 2 presents the timeline of the event study. We set the estimation period to 200 days and the event window to 61 days (30 days before and 30 days after the event date). We have set the trading day gap to 30 days to reduce the influence of the estimation period on the event window.

We define the event date as the date when the LPPLS confidence indicator exceeds a certain threshold. By definition, the LPPLS confidence indicator takes values between -1 to 1; the thresholds that we test for a positive LPPLS confidence indicator are 0, 0.1, 0.2...0.8, 0.9, 1. The thresholds that we test for a negative LPPLS confidence indicator are 0, -0.1, -0.2...-0.8, -0.9, -1.

To obtain more information on how the stock performs after a given event date, we also decompose both positive and negative LPPLS confidence indicators into different interval groups of (0, 0.1), (0.1, 0.2) ... (0.8, 0.9), (0.9, 1) and (0, -0.1), (-0.1, -0.2) ... (-0.8, -0.9), (-0.9, 1), respectively. All LPPLS confidence indicators failing to meet a chosen threshold or thresholds outside of a chosen interval were set to zero. When the LPPLS confidence indicator meets or goes beyond a chosen threshold or enters into a chosen interval, a bubble alarm signal (LPPLS confidence indicator) appears. Hence, we qualify those days as event dates. Moreover, we define a regime change as a clear price trend break.

Our null hypothesis (H0) and alternative hypothesis (H1) are:

H0: A regime change in the price does not occur after the event date.

H1: A regime change in the price occurs after the event date.

3.4. Data

For data on the Chinese market, we use the daily closing price of the China Securities Index Southwest Securities (CSI SWS) for 28 different industries, for the

period February 27, 2004 to July 23, 2020 (122,100 observations). The CSI SWS industry index contains the security prices of public companies in 28 industries listed in the Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange (SZSE). This index presents the capital-weighted aggregated price performance of different industries. We use the Wind database to obtain all the 28 industry sector indexes. The US dataset comprises the daily closing price of the US Morgan Stanley Capital International (MSCI) industry indexes. It represents the 24 industry groups in the US market; we use Thomson Reuters to obtain all the 24 industry indexes. Owing to data limitations, we could access the corresponding industry sector indexes from January 20, 2009, to July 23, 2020 (72,000 Observations).

We study the industry-level data for four reasons. First, in line with Greenwood et al. (2019), we implement our methodology for detecting bubbles using similar industry-level data. We agree with Greenwood et al. (2019) that most of the well-known historical bubbles have strong industry features. Second, industry-level data can provide more discriminatory statistical power to identify financial bubbles at a disaggregated level than at the market index level. In other words, we need more data to get a more generalized result. Third, we can compare one industry with others in the same period to obtain more detailed information about bubbles and to establish interconnections between industries in different markets. Fourth, industry-group level data are less influenced by an individual company's idiosyncratic risk, which makes the results less noisy.

To calculate the daily average abnormal returns for the US and Chinese events, we use the SSE composite index and S&P500 daily closing prices, with a corresponding time range as the market benchmarks. For the event study in China, we collect 1,131 positive and 383 negative LPPLS alarm signal events for the 28 industry groups in the Chinese market. For the event study in the US market, we collect 546 positive and 173 negative LPPLS alarm signal events for the 24 US industry groups. The details are presented in the Appendix.

4. Empirical Findings

In the following three subsections, we use both Chinese CSC SWS and MSCI US Industry groups' indexes to calculate the LPPLS confidence indicator. The duration t_2-t_1 of the time windows is scanned from 250 trading days to 30 trading days in steps of 1 trading day. We calculate the LPPLS confidence indicators for both positive (upward-accelerating price increases) and negative (downward-accelerating price decreases) bubbles. Subsequently, we present the cumulative average abnormal return (CAAR) for different LPPLS confidence signal thresholds and the LPPLS confidence signal intervals for the US and Chinese industry groups.

4.1 LPPLS Confidence Indicator for the US and Chinese Industry Groups

Figure 3 shows two time periods (in the middle of 2007 and 2015) during which the majority of the Chinese industry groups collectively showed strong positive LPPLS confidence indicators. It also shows two periods when several industry groups showed non-zero but smaller positive LPPLS confidence indicators (in late 2009 and early 2019). It shows three periods (late 2008, late 2012, and late 2018) when some of the Chinese industry groups concurrently had negative LPPLS confidence indicators.

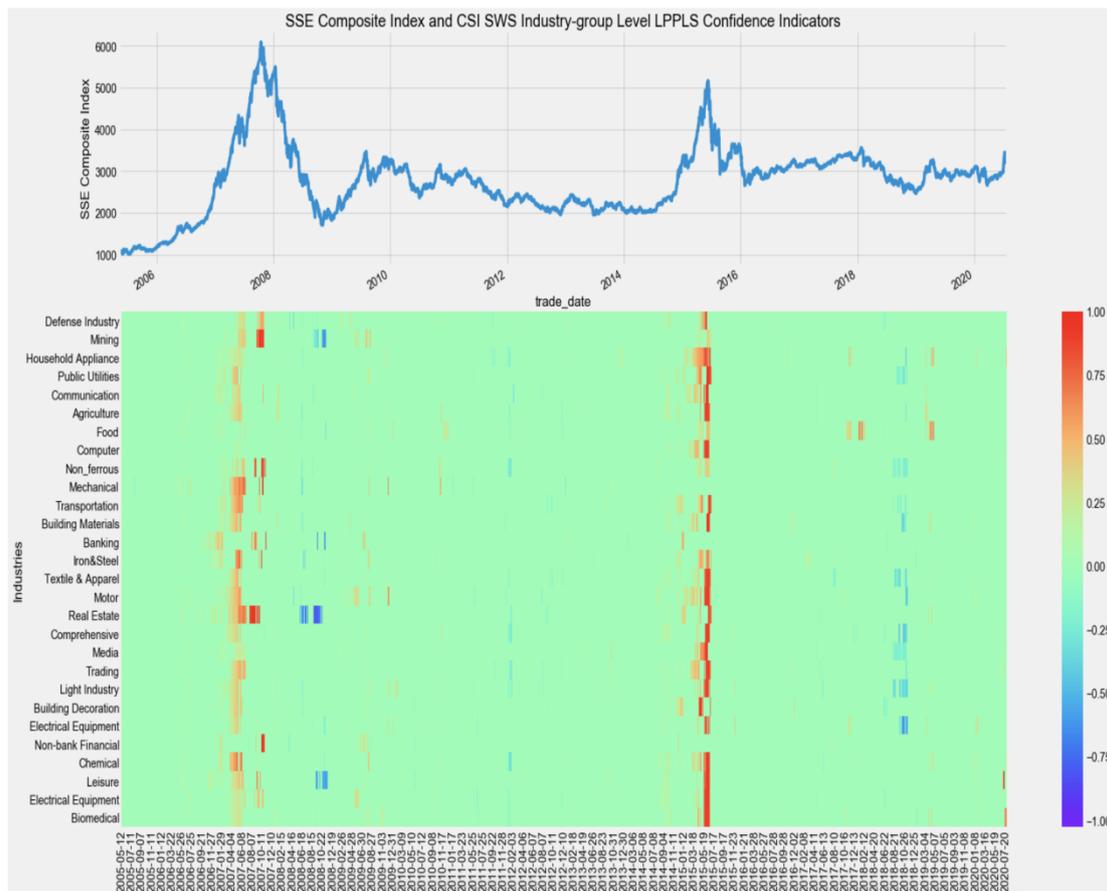


Figure 3. SSE Composite Index and LPPLS Confidence Indicator for the CSI SWS Industry Groups in thermal color scale given on the right. Hot (cold) colors correspond to positive (negative) bubbles. Sample period from 12/5/2005 to 23/7/2020.

By comparing the LPPLS Confidence Indicator patterns with the SSE composite index, it can be qualitatively seen that all of the industry-level bubble signals are followed by index-level regime changes—crashes or volatile sideways plateaus. This qualitative observation is quantified below by the event study.

Figure 4 shows that the LPPLS bubble signals for the US Industry groups do not exhibit the same patterns of collective clustering as those observed for the Chinese Industry groups. It shows that the US index rises much more consistently than that of the Chinese index. There are three time periods (early 2011, early 2013, late 2017) where many industries show moderate positive bubble alarm signals simultaneously. It also shows two time periods (late 2018 and early 2020) where some of the industries show negative bubble alarm signals. The first two periods of positive bubble clusters (early 2011 and early 2013) are not followed by significant crashes. However, a large

crash occurs after 15 industries together exhibit positive bubble alarm signals in early 2018.

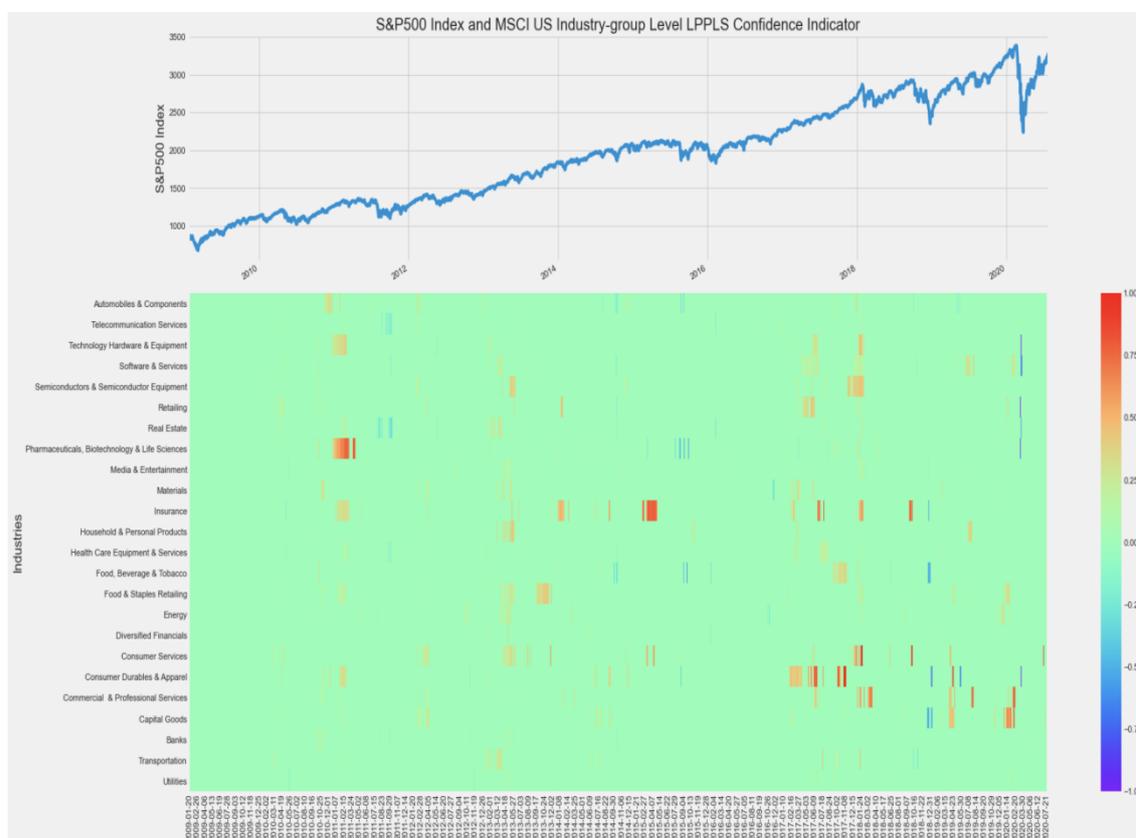


Figure 4. S&P500 Index and LPPLS (Positive and Negative) Alarm Signals for MSCI US Industry Groups in thermal color scale given on the right. Hot (cold) colors correspond to positive (negative) bubbles. Sample period from 12/5/2005 to 23/7/2020.

The LPPLS bubble alarm signals for the US and Chinese industry groups exhibit two different patterns. For the Chinese industry groups, the bubble signals are more concentrated and clustered in time. The prices of different Chinese industry groups also have quite similar bubble patterns, even though those industries have different business cycles. Conversely, for the different MSCI US industry groups, the bubble signals are more evenly distributed in time, though sometimes some of the industries collectively show milder bubble alarm signals.

4.2 Event Study for the LPPLS Confidence Indicators in the Chinese market

To quantify the value presented by the LPPLS confidence indicator to an investor, we apply the event study method. This helps us to analyze the market behavior before and after the events tagged by the LPPLS confidence indicator. The LPPLS confidence indicator is used to define a positive (negative) bubble event, called an *alarm* when (i) the LPPLS confidence indicator meets or crosses a pre-defined threshold or (ii) the LPPLS confidence indicator enters into the predefined interval groups. Case (i) corresponds to Figures 5, 6, and 8, while case (ii) corresponds to Figures 7 and 9. The numbers of events for the different thresholds and the two cases are given in Table 1 of the Appendix.

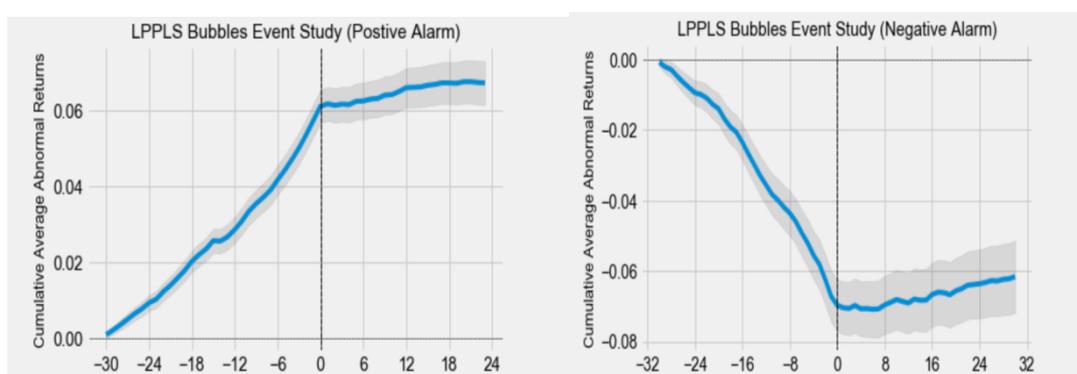


Figure 5. Plot of the Cumulative Average Abnormal Return (CAAR) for aggregated positive (left) and negative (right) LPPLS events from 30 days before to 30 days after each event for the Chinese CSI SWS Industry Groups from 2005 to 2020 described in section 3.4. The averages are performed over 1131 positive and 383 negative bubble alarm events, respectively, corresponding to the minimum threshold (condition to just be positive for positive bubbles and to be negative for negative bubbles). The Abnormal Return (AR) (3) is calculated by using the Market Model (2). The shadow area is the 95% confidence interval¹⁰ for the CAAR.

¹⁰ Confidence Interval = $\hat{r}_{i,t+e} \pm t_{CV} * se(\hat{r}_{i,t+e})$, where $\hat{r}_{i,t+e}$ is the average after-event return, t_{CV} is the critical value (1.96 represents a .95 level of confidence), $se(\hat{r}_{i,t+e})$ is the standard error of the regression and subscript (t+e) refers to the day of the event.

For 1,131 positive and 383 negative bubble alarm events identified in the Chinese CSI SWS industry groups from 2005 to 2020, Figure 5 shows the CAAR from 30 days before to 30 days after the event date. Both panels of Figure 5 demonstrate that, when an alarm is declared based on the LPPLS confidence indicator (for both positive and negative trends or bubbles), there is a clear change of regime from a strong increasing (decreasing) price acceleration to an approximate price plateau or rebound (for negative bubbles). In detail, the shadow area is the 95% confidence interval⁹ for the CAAR. The cumulative aggregated 30-day price abnormal return before the positive LPPLS confidence indicator event date grows by an average of 6% (a gain of 72% linearly annualized or 102% compounded), relative to the post-event cumulative aggregated average return of around 0.5% over the next 30 trading days. Similarly, for negative events identified by a negative LPPLS confidence indicator, the cumulative average abnormal return drops by 7% over the 30 trading days preceding the event date (a loss of 84% linearly annualized or 58% compounded), and the aggregated abnormal rebounds by around 1% over the 30 days following the event date. The general price behaviors of the outcomes of 1,131 positive alarm events and 383 negative alarm events in the Chinese market are consistent with the existing findings that the LPPLS model can detect regime shifts. The aggregated price dynamics around these events indicate that both positive and negative alarms can statistically predict the occurrence of changes of regime in the form of significant adjustments of price trends in Chinese industry-level data, which none of previous LPPLS-related research has covered.

Figure 6 records the (-30 days ,30 days) event window of price performance on the condition that the LPPLS confidence indicator meets different positive thresholds (defined in section 3.3). The graph indicates that, the higher the alarm thresholds, the larger is the increase in the average stock price within 30 days before the event date. Interestingly, after the event date, the outcomes exhibit two kinds of price trajectories. The green lines of the lower thresholds show that the price average will reach plateaus, while the red lines of the higher thresholds show that, the higher the signals, the more will be the number of precipitous crashes. Thus, we can conclude that the stronger the positive LPPLS confidence indicator (filtered by the thresholds), the larger is the confidence of a stronger unsustainable upward price acceleration (positive bubble), and the higher is the likelihood of a steep price decline thereafter (bubble crash). We

reconcile figures 5 and 6 by noting that the average presented in the left graph of Figure 5 is dominated by the lower thresholds shown in Figure 6, which are much more numerous and correspond to a transition to a plateau. For thresholds larger than 0.5, we observe a clear drawdown following the peak, which include 194 out of 1,131 events.

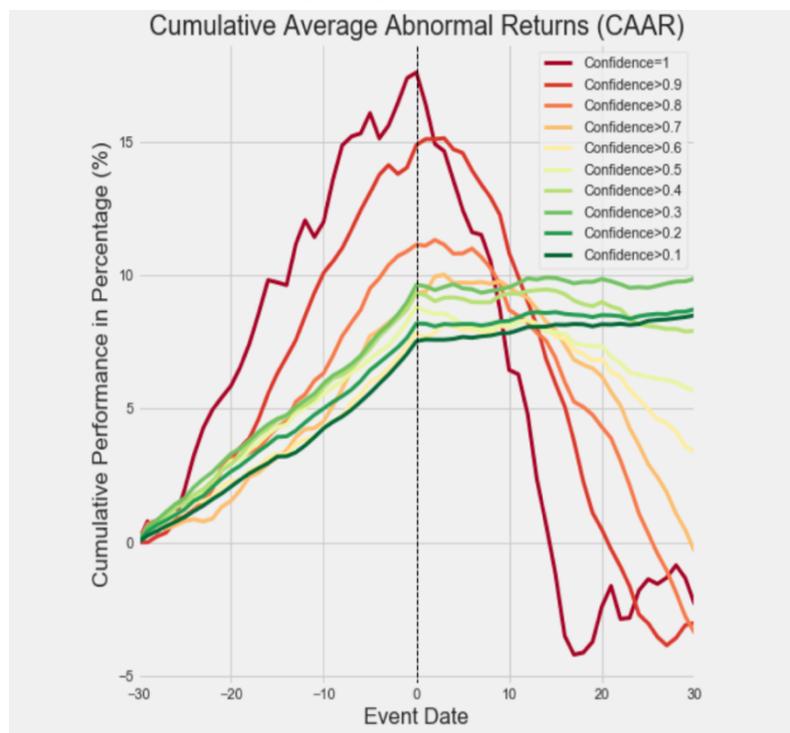


Figure 6. Plot of the Cumulative Average Abnormal Return (CAAR) for positive LPPLS events according to different LPPLS Positive Alarm Thresholds defined in section 3.3. The event window covers 30 days before to 30 days after each event for the Chinese CSI SWS Industry Groups from 2005 to 2020 described in section 3.4. The averages are performed over the corresponding subsets of a total of 1131 positive bubble alarm events corresponding to the different LPPLS Positive Alarm Thresholds (condition to just be positive bubbles). The number of events in each threshold class is given in Table 1. The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

Plot (a) of Figure 7 classifies the price performance for LPPLS confidence indicators that fall into different alarm intervals. This figure confirms the relationship between the level of the LPPLS confidence indicator and the extent of post-event price performances—the larger is the association between the positive LPPLS confidence and fiercer increases, the more aggressive are the price declines. Conversely, the smaller positive LPPLS confidence indicator does not necessarily predict crashes; instead, the average price performance might still have an upward trend, but the speed

of the price increase clearly drops after the event date. In particular, the strongest positive LPPLS confidence indicator average shows a more than 17% relative return drop in around 15 trading days (leading to a 283% annual linear loss) in the Chinese market.

Plot (b) of Figure 7 presents a sanity check that our results are not just rediscovering the standard short term reversal effect (Jegadeesh, 1990). It shows the Cumulative Average Abnormal Return (CAAR) obtained from 1600 randomly selected events. Specifically, we form eight groups of 200 events, each group defined by a given return interval in the first 30 day period. These return intervals are 2-4%, 4-6%, 6-8%, ..., 16-18%. In other words, for a group defined by a given return interval, say 8-10%, we pick 200 random times such that the CAAR of the Chinese stock market CSI SWS Industry Groups over the interval from 30 days preceding each random time to this random time is found between 8 and 10%. The curves in panel (b) of Figure 7 show the CAAR averaged over the 200 random events in each group. The times from -30 days to 0 are before the random event times. The positive times from 0 to 30 days correspond to the post (random) event times. Comparing with Plot (a), the CAAR of the randomly selected events in Plot (b) show a clear momentum effect, instead of the impressive change of regimes in the form of transitions from strongly increasing prices to large drawdowns detected for the larger LPPLS Confidence Indicators in Plot (a).

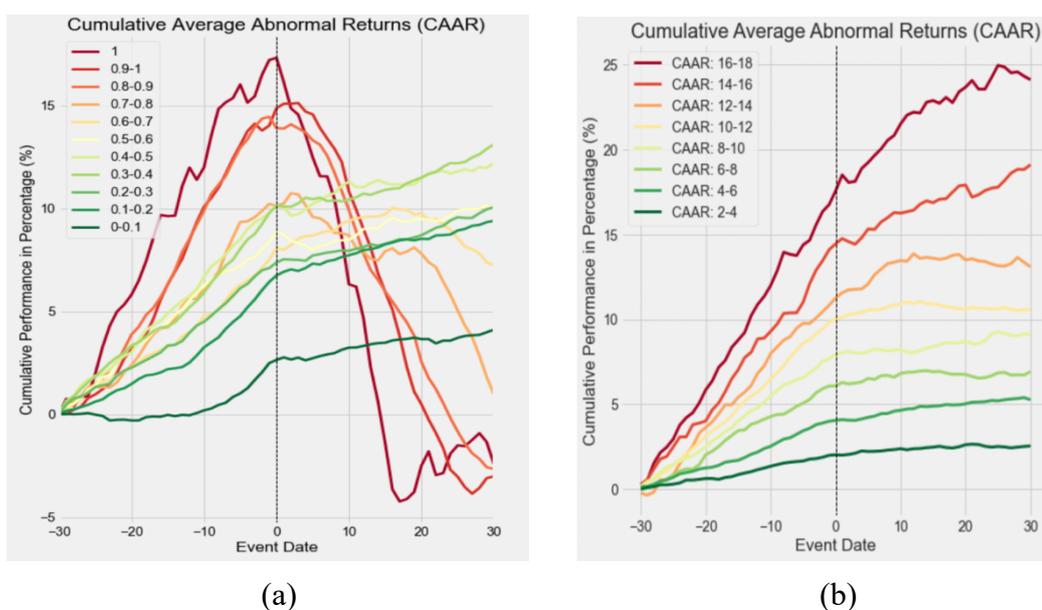


Figure 7. Plot (a) presents the Cumulative Average Abnormal Return (CAAR) for 1131 positive bubble alarm events corresponding to the different LPPLS Positive Alarm Intervals defined in

section 3.3. The number of events in each interval class is given in Table 1. As a comparison, plot (b) presents for 1600 randomly selected events the corresponding 30-day Cumulative Average Abnormal Return (CAAR). The data is the Chinese CSI SWS Industry Groups from 2005 to 2020 described in section 3.4. The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

We also tested the negative LPPLS confidence indicator with different thresholds. The result shows that the price behavior is noisier than for the positive LPPLS confidence indicator. Figure 8 shows that the most negative LPPLS confidence indicator values are followed by more volatile cumulative returns after the event date. Conversely, the mildly negative LPPLS confidence indicator values are followed by more stable price plateaus.

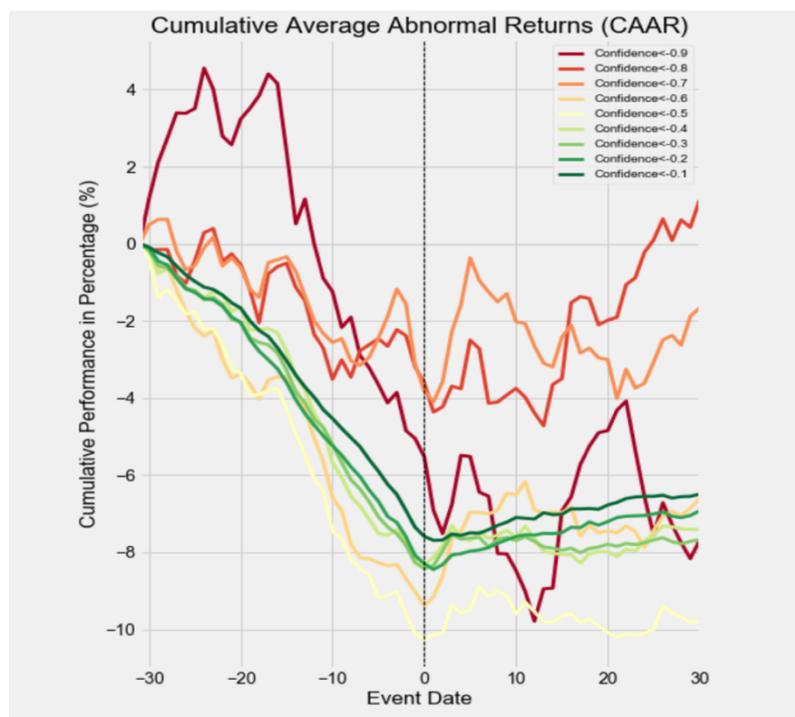
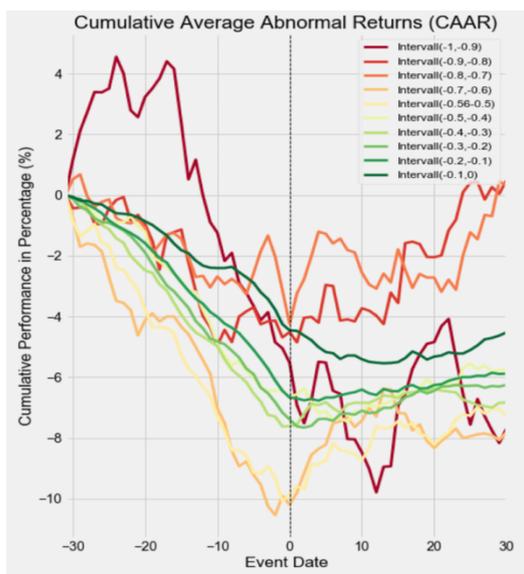


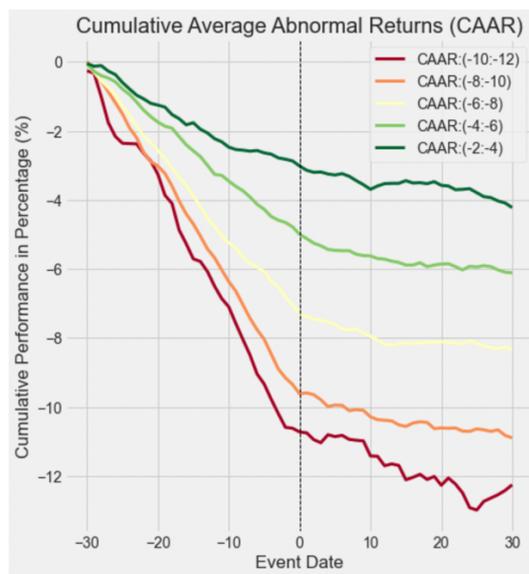
Figure 8. Plot of the Cumulative Average Abnormal Return (CAAR) for negative LPPLS events according to different LPPLS Negative Alarm Thresholds defined in section 3.3. The event window covers 30 days before to 30 days after each event for the Chinese CSI SWS Industry Groups from 2005 to 2020 described in section 3.4. The averages are performed over the corresponding subsets of a total of 383 negative bubble alarm events corresponding to the different LPPLS Negative Alarm Thresholds (condition on being negative bubbles). The number

of events in each threshold class is given in Table 1. The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

We also classify the negative LPPLS confidence indicator into different intervals. Relative to Figure 8, Figure 9 presents clearly that the stronger the negative alarm signals before the peak, the more volatile is the price after the peak. It must be noted that the patterns observed for the negative alarm signal results do not mirror those obtained for the positive alarm signal results mentioned earlier (the higher positive LPPLS confidence indicators are followed by large crashes). The larger negative LPPLS confidence indicator values do not lead to strong rebounds; instead, they lead to a higher level of volatilities. Notwithstanding this difference, there is clear evidence that the LPPLS negative alarm signals clearly detect well-defined regime changes.



(a)



(b)

Figure 9. Plot (a) presents the Cumulative Average Abnormal Return (CAAR) for 383 positive bubble alarm events corresponding to the different LPPLS Positive Alarm Intervals defined in section 3.3. The number of events in each interval class is given in Table 1. As a comparison, plot (b) presents for 1000 randomly selected events the corresponding 30-day Cumulative Average Abnormal Return (CAAR). The data is the Chinese CSI SWS Industry Groups from 2005 to 2020 described in section 3.4. The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

Plot (b) of Figure 9 is the analog of Plot (b) of Figure 7 for large negative CAAR over the pre-event time interval. It shows the Cumulative Average Abnormal Return (CAAR) obtained from 1000 randomly selected events. Specifically, we form five groups of 200 events, each group defined by a given return interval in the first 30 day period. These return intervals are -2 to -4%, -4 to -6%, -6 to -8%, -8 to -10%, -10 to -12%. In other words, for a group defined by a given return interval, say -8 to -10%, we pick 200 random times such that the CAAR of the Chinese stock market CSI SWS Industry Groups over the interval from 30 days preceding each random time to this random time is found between -8 and -10%. The curves in panel (b) of Figure 9 show the CAAR averaged over the 200 random events in each group. The times from -30 days to 0 are before the random event times. The positive times from 0 to 30 days correspond to the post (random) event times. Comparing with Plot (a) of Figure 9, the CAAR of the randomly selected events in Plot (b) exhibit a momentum effect, rather than the rebound and increased volatilities detected for the larger LPPLS Confidence Indicators in Plot (a).

4.3 Event Study for the LPPLS Confidence Indicators in the US market.

Since the evidence presented in the previous section strongly rejects the null hypothesis and thus supports the hypothesis that the LPPLS confidence indicator can detect regime changes in China (a typical emerging market), we investigate whether the LPPLS confidence indicator can detect regime changes occurring in the US market—a developed market. We apply the same method, as in section 4.2, by setting positive and negative LPPLS confidence thresholds and interval groups and event dates, as defined in section 3.3. Our method is as follows: (i) the LPPLS Confidence indicator meets or

crosses the LPPLS event alarm thresholds defined in section 3.3, and (ii) the LPPLS confidence indicators enter into different LPPLS event alarm intervals defined in section 3.3. Figures 10, 11, and 13 belong to (i), and Figures 12 and 14 belong to (ii). The number of events in these different classes is given in Table 2 of the appendix.

For the 24 MSCI US industry groups from 2009 to 2020, all 546 positive LPPLS alarm signal event studies give a CAAR before the LPPLS alarm event of 2.5% over 30 trading days. After the event data, the price level reached a plateau, with a potential downward trend, as shown in Figure 10. Conversely, for the 173 negative bubble events, the pre-event 30-day showed a 5% price downward CAAR. After the event date, the price bounced back a little and had an upward potential trend, as shown in figure 10. Hence, for the US market, the LPPLS confidence indicator can also be used to predict regime changes. Notably, the aggregated positive bubbles in the US market are smaller than those in the Chinese market.

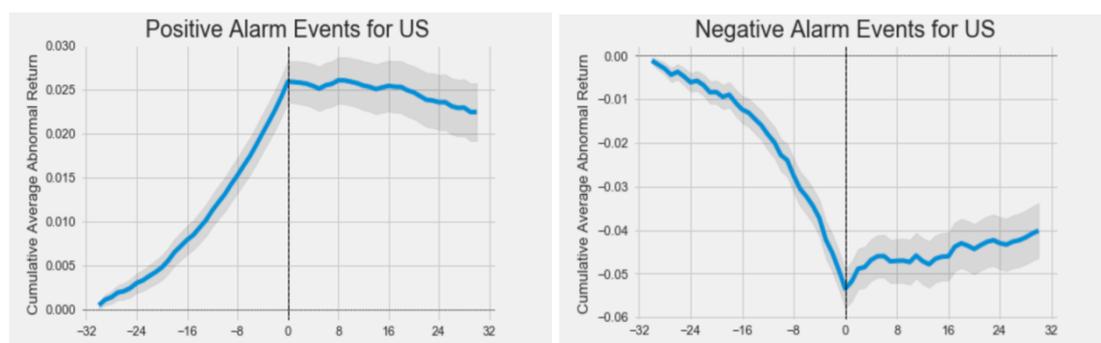


Figure 10. Plot of the Cumulative Average Abnormal Return (CAAR) for aggregated positive (left) and negative (right) LPPLS events from 30 days before to 30 days after each event for the MSCI US Industry Groups from 2009 to 2020 described in section 3.4. The averages are performed over 546 positive and 173 negative bubble alarm events, respectively, corresponding to the minimum threshold (condition to just be positive for positive bubbles and to be negative for negative bubbles). The Abnormal Return (AR) (3) is calculated by using the Market Model (2). The shadow area is the 95% confidence interval for the CAAR.

Following the same methodology as for the Chinese market with classes of events defined by different threshold values of the confidence indicator, Figure 11 shows that the larger the LPPLS thresholds, the larger is the subsequent drop of the price trajectory. However, the US market's CAAR and the relative crashes after the event date are not as large and severe, respectively, as those of the Chinese market.

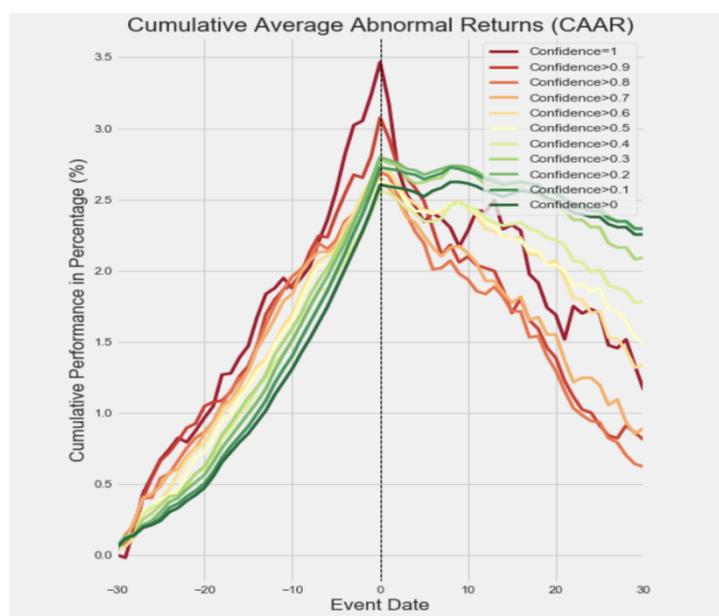


Figure 11. Plot of the Cumulative Average Abnormal Return (CAAR) for positive LPPLS events according to different LPPLS Positive Alarm Thresholds defined in section 3.3. The number of events in each interval class is given in Table 2. The event window covers 30 days before to 30 days after each event for the MSCI US Industry Groups from 2009 to 2020 described in section 3.4. The averages are performed over the corresponding subsets of a total of 546 positive bubble alarm events corresponding to the different LPPLS Positive Alarm Thresholds (condition to just be positive bubbles). The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

We also divide the alarm signals into different intervals. Plot (a) of Figure 12 shows results similar to those in Plot (a) of Figure 7 for the Chinese market. On average, the larger positive LPPLS confidence indicator values correspond to sharp increases and steep crashes, while smaller LPPLS confidence indicator values correspond to a transition from an increased price to a plateau.

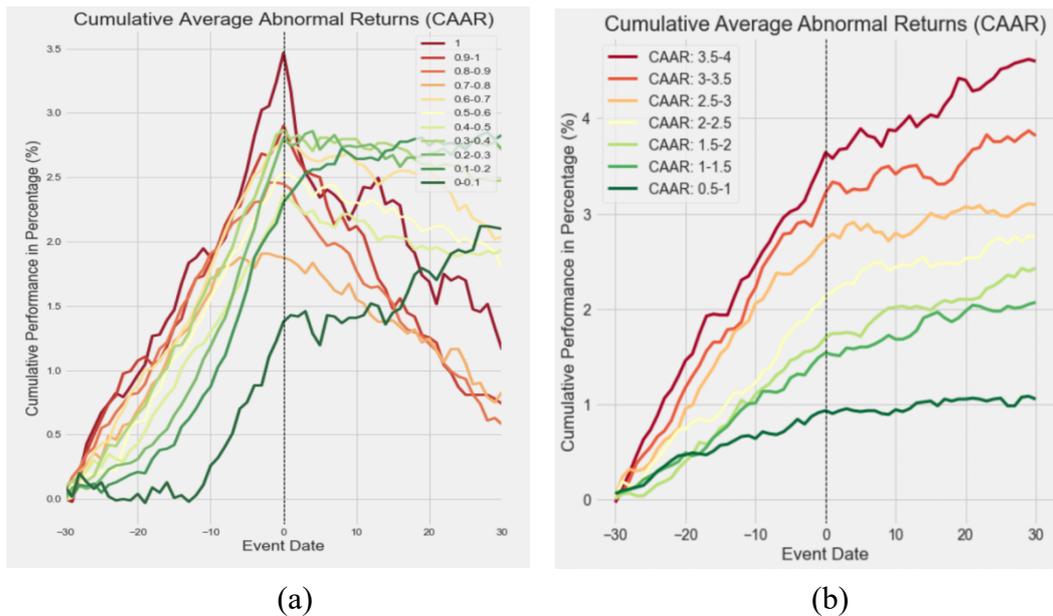


Figure 12. Plot (a) presents the Cumulative Average Abnormal Return (CAAR) for 546 positive bubble alarm events corresponding to the different LPPLS Positive Alarm Intervals defined in section 3.3. The number of events in each interval class is given in Table 2. As a comparison, plot (b) presents for 700 randomly selected events the corresponding 30-day Cumulative Average Abnormal Return (CAAR). The data is the MSCI US Industry Groups from 2009 to 2020 described in section 3.4. The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

Plot (b) of Figure 12 is the same as Plot (b) of Figure 7, but with different return intervals that match the different return amplitudes of the US stock market. It shows the Cumulative Average Abnormal Return (CAAR) obtained from 700 randomly selected events. Specifically, we form seven groups of 100 events, each group defined by a given return interval in the first 30 day period. These return intervals are 0.5-1.0%, 1.0-1.5%, ..., 3.5-4.0%. For a group defined by a given return interval, say 3.0 to 3.5%, we pick 100 random times such that the CAAR of the US stock market Industry Groups over the interval from 30 days preceding each random time to this random time is found between 3 and 3.5%. The curves in panel (b) of Figure 12 show the CAAR averaged over the 100 random events in each group. The times from -30 days to 0 are before the random event times. The positive times from 0 to 30 days correspond to the post (random) event times. Comparing with Plot (a) of Figure 12, the CAAR of the randomly selected events in Plot (b) show a momentum effect, which is clearly distinct from the

large drawdowns following the bubble regimes detected by larger LPPLS Confidence Indicators in Plot (a).

Testing the negative LPPLS thresholds in the United States, we find that the negative LPPLS confidence indicator is particularly suited for detecting regime changes in this country. Figure 13 shows that, after the event date, the index prices bounce up immediately. Similar to the Chinese market, the higher the amplitude of the negative LPPLS Confidence Indicator, the more volatile will be the post-event price performance.

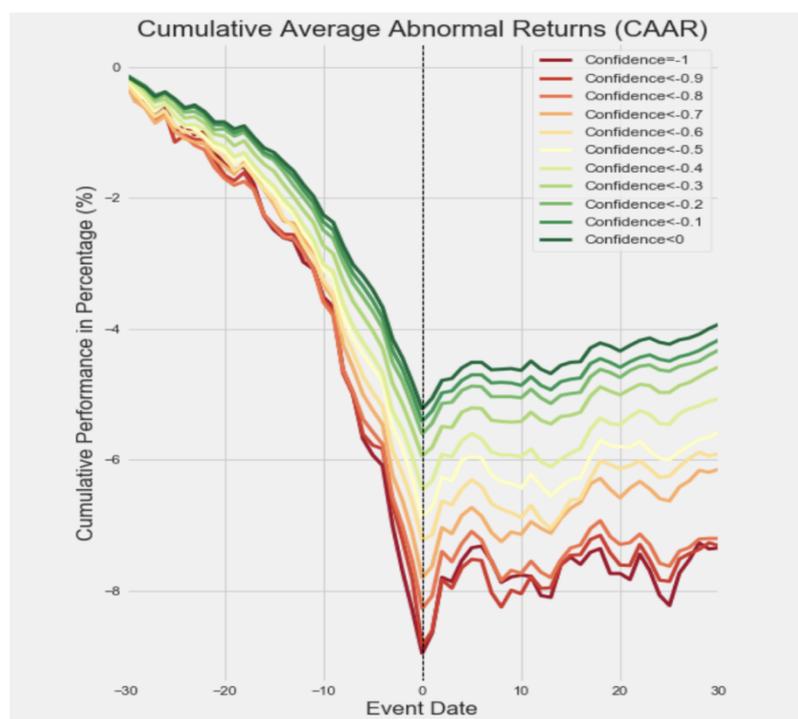
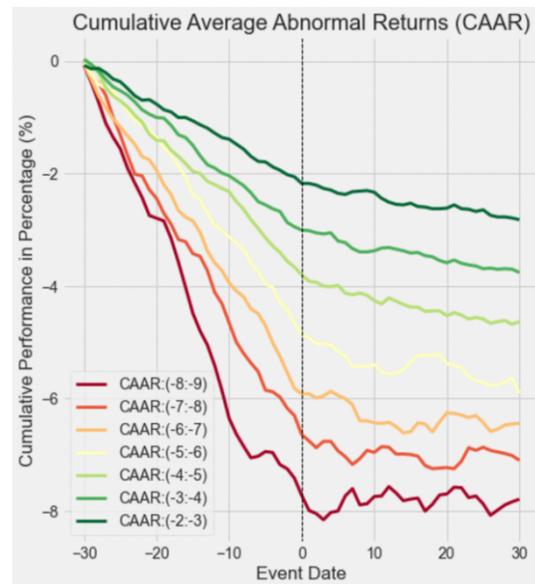
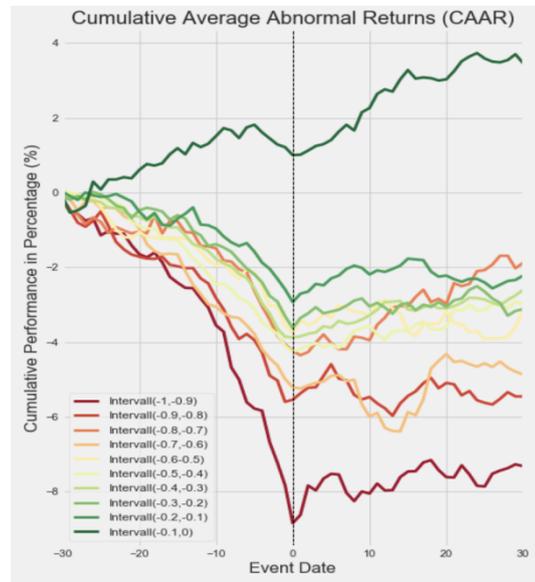


Figure 13. Plot of the Cumulative Average Abnormal Return (CAAR) for negative LPPLS events according to different LPPLS Negative Alarm Thresholds defined in section 3.3. The number of events in each interval class is given in Table 2. The event window covers 30 days before to 30 days after each event for the MSCI US Industry Groups from 2009 to 2020 described in section 3.4. The averages are performed over the corresponding subsets of a total of 173 negative bubble alarm events corresponding to the different LPPLS Negative Alarm Thresholds (condition on being negative bubbles). The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

Plot (a) of Figure 14 shows that the stronger the negative LPPLS confidence indicator values, the faster are the price declines before the event date and the more sudden are the rebounds. Moreover, the larger negative bubble signals are associated with a higher level of the volatility after the event date.



(a)

(b)

Figure 14. Plot (a) presents the Cumulative Average Abnormal Return (CAAR) for 173 positive bubble alarm events corresponding to the different LPPLS Positive Alarm Intervals defined in section 3.3. The number of events in each interval class is given in Table 2. As a comparison, plot (b) presents for 700 randomly selected events the 30-day Cumulative Average Abnormal

Return (CAAR). The data is the MSCI US Industry Groups from 2009 to 2020 described in section 3.4. The Abnormal Return (AR) (3) is calculated by using the Market Model (2).

Plot (b) of Figure 14 is the analog of Plot (b) of Figure 9 for large negative CAAR over the pre-event time interval. It shows the Cumulative Average Abnormal Return (CAAR) obtained from 700 randomly selected events. Specifically, we form seven groups of 100 events, each group defined by a given return interval in the first 30 day period. These return intervals are -2 to -3%, -3 to -4%, ..., -8 to -9%. For a group defined by a given return interval, say -6 to -7%, we pick 100 random times such that the CAAR of the US stock market Industry Groups over the interval from 30 days preceding each random time to this random time is found between -6 and -7%. The curves in panel (b) of Figure 12 show the CAAR averaged over the 100 random events in each group. The times from -30 days to 0 are before the random event times. The positive times from 0 to 30 days correspond to the post (random) event times. Comparing with Plot (a) of Figure 14, the CAAR of the randomly selected events in Plot (b) exhibit a momentum effect, in clear contrast with the rebound and increased volatility effect detected by larger LPPLS Confidence Indicators in Plot (a).

4.4 Comparison between the US and Chinese markets

The evidence presented in previous sections for both the US and Chinese markets strongly rejects the null hypothesis that no regime change can be detected and supports the alternative hypothesis that the LPPLS confidence indicator can predict regime changes around event dates corresponding to market peaks. The model detects clear breaks in the price acceleration before the event date, followed by a correction or plateau after the event date, depending on the amplitude of the LPPLS confidence Indicators. The pre-event price accelerations of positive bubbles in the US market are smaller than those in the Chinese market. The industry groups in the Chinese market tend to have synchronized LPPLS confidence indicator values, even though each industry has a different cycle. This phenomenon suggests that the different trends observed in the Chinese market are the result of the collective behavior of several

industries. This finding implies stronger herding behaviors in the whole market developing in synchrony.

We have used the thresholds and intervals to decompose and categorize the LPPLS confidence indicator in different event classes. The analysis shows that, for positive LPPLS confidence indicators in both the US and Chinese markets, larger thresholds and interval values are associated with severe price drops within a very short period (a crash) after the event date. However, the price after the event date for lower quantile groups tends to reach a plateau. We conclude that the *crashes* that economists would like to characterize are associated with the interval groups with the larger positive LPPLS Confidence indicators.

For negative bubbles in both the US and Chinese markets, the magnitudes of the bounce-backs following the event date are mild, which breaks the symmetry with the price patterns documented for positive bubbles. Moreover, we observe that price volatilities increase in both the US and Chinese markets after the event date.

4.5 Interpretation of the two classes of post-event dynamics: Overreaction and Underreaction

Figure 15 summarizes our main findings on how an accelerating price trajectory ends. Specifically, we identify two classes of regime shifts—(i) super-exponential price growth followed by a crash (genuine bubble) and (ii) super-exponential price growth followed by a plateau, suggesting a convergence to a relatively stable price level (apparent bubble).



Figure 15. Simplified representation of the two regime shifts identified in the study of accelerated price trajectories in the Chinese and US markets. Left: a genuine bubble followed

by a crash or large drawdown. Right: apparent bubble with strong price growth culminating in a plateau, suggesting a new consensus for the underlying fundamental value.

In the genuine bubble case (left panel of figure 15), the price overshoots and subsequently corrects with a drawdown. Specifically, after the price accumulates a large deviation from its implicit fundamental value (i.e., the stable price level), selling orders may suddenly synchronize, while there is an exhaustion of available cash for buying orders. Together, the large synchronized sell-out orders and the absence of buyers lead to a cascading decline in prices. This synchronization behavior can be detected by the LPPLS model as shown above (Sornette and Cauwels, 2015a).

In the apparent bubble case (right panel of figure 15), the accelerating price should be interpreted as, in fact, a *slow* convergence (even if accelerating) to a stable price level. Investors may misinterpret this price acceleration as a genuine bubble and short it. However, this would be unsuccessful, owing to the progressively growing consensus on the underlying value of the assets. The plateau, following the break in the acceleration phase, embodies the new equilibrium associated with all available information that has finally been *digested* by the investors.

In an efficient market, a stock price ought to perfectly adjust to a new level (almost) immediately after new information comes to the market (Fama, 1970). However, in line with a large body of literature presenting evidence against the EMH theory, the speed and magnitude of a price adjustment are not always efficient, and the market is, to some extent, predictable owing to structural and behavioral reasons. In addition, arbitrage opportunities can exist for a long-time in the presence of frictional costs and risks. Our empirical result strongly rejects the EMH that the current price reflects all of the information. Our finding suggests that it is possible to predict the market price patterns based on the past price trajectory, at least in pockets of predictability associated with large amplitudes of the LPPLS confidence indicator.

Our finding of the two types of regime changes is somewhat related to the overreaction hypothesis and underreaction hypothesis. Overreaction indicates that investors tend to be excessively optimistic about new information, and hence stock prices tend to go beyond their true value and are followed by subsequent corrections. This is exemplified by the genuine bubble regime shown in the left panel of Figure 15.

Underreaction, which always leads to price momentum or post-earnings announcement price drift, implies that stock prices change less than their true value justified by the news (De Bondt and Thaler, 1985). This leads to delays in the price adjustment processes, as illustrated in the right panel of Figure 15.

We have found that larger LPPLS confidence indicator values (implying longer super-exponential price trajectories) statistically lead to faster and larger price corrections, which suggests an overreaction. In contrast, smaller LPPLS confidence indicator values (indicating shorter super-exponential price trajectories) are associated with a price plateau after the event date. This shows that the LPPLS confidence indicator can detect regime changes both in the presence of overreaction and underreaction, according to the strength of the indicator. Thus, we can conclude that the LPPLS confidence indicator can *unify* the overreaction and underreaction hypotheses, converging to a similar conclusion as that of Hong and Stein (1999), while using different models.

Few academic studies have provided behavioral explanations of overreaction and underreaction. They suggest that overreaction comes from herding, representativeness, and overconfidence. They attribute underreaction to anchoring, conservatism, the disposition effect, and the slow diffusion of information (Antonacci, 2016).

In the case of overreaction, investors tend to herd when information is scarce or not easily available (Bikhchandani and Sharma, 2000). It is boundedly rational to imitate in the absence of sufficient information (Roehner and Sornette, 2000). Young male investors with lower portfolio values and less educated investors exhibit more overconfidence, and thus display more irrational behavior (Tekce and Yilmaz, 2015). Representativeness, as defined by Tversky and Kahneman (1974), refers to the fact that investors believe that the history of a remarkable performance of a given firm is *representative* of the firm performance and that this will continue by extrapolation. Hence, overreaction can be a part of the explanation for the occurrence of bubbles. The literature discusses several other mechanisms (e.g., Kaizoji and Sornette, 2010; Brunnermeier and Oehmke, 2013; Xiong, 2013; Sornette and Cauwels, 2015b).

Concerning underreaction, investors are influenced by the initial price as the reference point, which leads to adjustments of their usually insufficient estimation. This is known as the anchoring effect (Hong and Stein, 1999). This underreaction is also

attributed to conservatism, which means that people tend to cling to their prior views at the expense of acknowledging new information (Kahneman and Tversky, 1979). The disposition effect, proposed by Shefrin and Statman (1985), holds that investors are likely to sell the profit-making stocks while holding the loss-making stocks. The theory on the slow diffusion of information holds that the slow diffusion of information and interaction between investors can explain price underreaction and overreaction in the short- and mid-runs, respectively (Hong and Stein, 1999). While possibly part of the explanation of the dynamical development of apparent bubbles, these effects may not fully elucidate why the price plateaus at the end of the accelerated price phase. In an apparent bubble, the market participants progressively convince themselves that the new correct price is higher. However, this self-convincing takes a long time (weeks, months, or years) and likely involves imitation, herding, and various feedback loops¹¹, together with some form of fundamental information that anchors the price to its final plateau.

4.6 Leverage effect for the negative LPPLS Confidence Indicators

Following a large price appreciation, based on a strong reading of the positive LPPLS confidence indicator, the price corrects in a crash or large drawdown. Following a large price depreciation, based on a strong reading of the negative LPPLS confidence indicator, the price does not rebound as it would if there were symmetry between upward price acceleration for positive bubbles and downward price acceleration for negative bubbles. Conversely, a large price depreciation, based on a strong reading of the negative LPPLS confidence indicator, is followed by an increase in its volatility, and not so much by a rebound (which would be the symmetric shape to a crash).

Two explanations can be advanced for this increased volatility following a strong price decline associated with a negative bubble. The leverage effect, first proposed by Black (1976), might be part of the explanation. When asset prices decline, companies become more leveraged as the ratio of their debt value over equity rises. This increases the leverage of the firms' capital structures. The increased leverage deteriorates the

¹¹ It is similar to the positive feedback investment strategies proposed by DeLong et al. (1990).

financial state of public companies and, consequently, increases the systematic risk of common stocks. Consequently, the cost of capital becomes larger to reflect the higher risk of financial insolvency, generating a volatility feedback effect (Campbell and Hentschel, 1992). Thus, declines in stock prices are expected to be accompanied by increases in volatility (Nelson, 1991 Engle and Ng, 1993). However, this explanation may be insufficient to explain the observed increases in volatility after a price decline. Other mechanisms, including behavioral ones, may play a significant role (Figlewski and Wang, 2000; Bouchaud, Matacz, and Potters, 2001).

A second explanation may be captured by the statement that “misfortunes never come singly.” Ding, Giesecke, and Tomecek (2009) note the existence of a cross-sectional self-exciting behavior of volatility in the sense that the default or the large negative shock to one company, cross-sectionally, tends to increase the likelihood of default or large downward movements of other companies (see also Siczka, Sornette, and Holyst, 2011, Smug et al., 2022). Overall, such cross-sectional propagation leads to a self-exciting pattern in the market and, therefore, volatility spillovers from one company to others. Thus, investors are likely to observe that one episode of market turmoil increases the chance of another subsequent turmoil (Azizpour, Giesecke, and Schenkler, 2018; Smug et al., 2022). Accordingly, when bad news is initially released to the public, the price might drop. In such a scenario, while some investors might sell the stock owing to stop-loss orders, margin calls, portfolio insurance, or regulatory constraints, others may buy the cheaper stock. Overall, there is an increase in transaction volume, translating into an increase in volatility (Gabaix, Gopikrishnan, Plerou, and Stanley, 2003). In the presence of the self-exciting behavior mentioned above, the initial negative shock, like the *tip of the iceberg*, might lead to more bad news and further increase the volatility of the stock price¹².

5. Conclusion

We applied the event study method to study systematically the LPPLS (log-periodic power law singularity) confidence indicators. The daily closing prices of the

¹² An illustration is Lehman Brothers' closure on September 15, 2008.

Chinese CSI SWS industry groups and MSCI US industry groups were used to identify positive and negative bubbles. The event study method allowed us to statistically characterize the pre- and post-event price behaviors of positive and negative bubbles.

Our main result strongly supports the hypothesis that the LPPLS confidence indicator can detect regime changes. Thus, it shows that the market is inefficient and future price patterns can be predicted based on the past price information, at least in pockets of predictability associated with the ends of financial bubbles.

We reached three important conclusions. First, based on extensive event studies in both the US and Chinese markets, we found that the LPPLS framework can systematically detect unsustainable price increases and decreases (including bubbles) with only price data, ex-ante and causally. In this manner, we refuted the claims by Fama (2014) and Greenwood et al. (2019) that bubbles cannot be identified in real time.

Second, we found bubbles at the industry-group level in both China and the US markets. We showed that a smaller LPPLS confidence indicator can detect a short-term continuation, while a larger indicator can detect a strong reversal. Particularly, the Chinese market showed stronger collective bubbles (followed by predictable price decline) that develop in different industries simultaneously, despite their different business cycles. However, stronger bubbles in the US market (followed by a largely predictable price decline) are relatively evenly distributed in different industry groups over time, suggesting more decoupling. Moreover, both positive bubbles and crashes are significantly more extreme in the Chinese market than that in the US market.

Third, positive and negative bubbles are not symmetric, and the alarms obtained from the LPPLS confidence indicator do not always diagnose the occurrence of subsequent crashes. For positive bubbles, larger positive LPPLS confidence indicator values diagnose an overactive price behavior, leading to a peak followed by a crash. In contrast, a smaller LPPLS confidence indicator value predicts a regime change to a less extreme regime—just a plateau breaking the preceding accelerating price. For negative bubbles, the US market shows a clearer rebound behavior following the price trough, relative to the Chinese bubbles. However, in both markets, the larger the amplitude of the negative alarm signals, the more volatile is the price after the critical time of the price peak.

Finally, we conclude that a simple classification of price regimes based on the LPPLS confidence indicator can predict price regime switching in both the US and Chinese. Our model can also *unify* the overreaction and underreaction phenomena. Particularly, strong positive LPPLS confidence indicators predict a strong price decline following a large price appreciation, which, according to Fama's definition, is a bubble.

Appendices

Appendix A: The construction of the LPPLS Confidence Indicator

We use equation (1) to detect financial bubble patterns in financial price time series, by fitting the model to data. Naturally, we need to check the goodness of each fit, that is, whether a fit qualifies a bubble signal. Moreover, since we are dealing with noisy financial data, it is better to do multiple fits at each time point to build a strong indicator, than simply to do just one fit. Based on these ideas, we used the LPPLS Confidence Indicator. We now discuss in detail the construction of the LPPLS Confidence Indicator, especially the two most important aspects of it, namely the goodness of fit and the multiple fits at each time point.

An important ingredient of the LPPLS Confidence Indicator is the manner of qualifying a good fit of the LPPLS model. The first thing one can imagine is the loss function, for instance, the mean squared error (MSE) between each element in the input log prices and the corresponding LPPLS fits. Thus, our first criterion of a good LPPLS fit is based on the MSE loss. In other words, if the MSE of a fit is larger than a threshold, the fit is considered bad, that is, it does not qualify a bubble signal. However, that does not mean that a low MSE fit alone qualifies as a good fit, because a bubble pattern has its unique characteristics, which leads to our next criterion for good LPPLS fits.

Our second criterion is the relative closeness of t_c to t_2 , where t_c is one of the fitted parameters of the LPPLS model, indicating the time that a bubble, if it exists, will become unsustainable. Here, t_2 is the timestamp of the last data point in the price time series used to fit the model. The calibration is performed on the log-price data in a time window starting from the start time t_1 to the current time t_2 . If t_c is far away from t_2 , in the sense that $|t_c - t_2|/|t_2 - t_1|$ is larger than a threshold, the fit does not qualify as a bubble signal.

For the third criterion, we first introduce an indicator called damping, which is defined as $|B_m|/|C\omega|$, and denotes the intensity of a bubble, if it exists. The indicator comes from the fact that, in the JLS rational bubble expectation formulation (Johansen et al., 1999; 2000), the derivative of the expected log-price is proportional to the crash hazard rate, and thus must remain positive by definition of a probability. The condition for this to hold is $|B_m|/|C\omega| > 1$ (Bothmer and Meister, 2003). This motivates us to define the “damping” parameter $|B_m|/|C\omega|$ and use its quantitative values for classification (while not imposing the strict lower threshold of 1). A good LPPLS fit is then such that the damping parameter is larger than a threshold.

The fourth criterion concerns parameter m , which should be between 0 and 1. This is to describe that, in a bubble phase, the prices usually accelerate before they correct. A value $m=1$ would correspond to a linear average log-price increases (i.e., an average exponential growth of the price) and values $m>1$ correspond to concave decelerating expected log-prices.

The fifth criterion concerns the number of oscillations of the fitted model, which is required to be larger than some threshold. If there are not so many oscillations, the probability that a bubble is mature or even exists is interpreted as low. The number of oscillations can be easily computed from the parameters of the LPPLS model, giving $\omega \log(|t_2 - t_c| / |t_1 - t_c|) / 2\pi$.

When a fit passes all the above criteria, it qualifies as a bubble signal. At the same time, we need to check the sign of parameter B . If B is positive, it detects a negative bubble and the confidence indicator takes a negative sign; otherwise, it detects a positive bubble, and the confidence indicator takes a positive sign.

Another ingredient of the LPPLS Confidence Indicator involves multiple fits. As discussed above, for a time series from t_1 to t_2 , we fit the log-prices in a series of historical time windows ending at t_2 , which are $[t_2-30, t_2]$, $[t_2-35, t_2]$, ..., $[t_1, t_2]$. We usually use a step size of 5 days to define increasing time window lengths from a minimum of 30 days to a maximum equal to $t_2 - t_1$. This is a compromise between having sufficient statistics and robustness over many time windows while keeping computation efforts to a reasonable level. When we fit the log-price in all these time windows, we apply the aforementioned criteria to filter out fits that do not qualify as reliable bubble signals.

The LPPLS Confidence Indicator is thus the fraction of good fits among the fits spanning all time windows. We compute the median value of damping indicators of these qualified fits, which quantifies the severity of the bubble. It is clear that a large LPPLS Confidence indicator indicates a stably growing bubble found consistently across many time windows of increasing sizes, while a high median damping indicator of qualified fits indicates that the bubble has a significant amplitude.

The process of constructing the LPPLS Confidence Indicator can be summarised as follows:

1. We determine the size (number of time stamps) of the input time series, denoted by $T=t_2-t_1$ which is a fixed meta parameter, the threshold for the MSE denoted by err , the threshold for the closeness of t_c to t_2 denoted by t_{cd} , the threshold for the damping parameter denoted by

d , the threshold for the number of oscillations denoted by osc .

2. For a price series from t_1 to t_2 , we fit the LPPLS model and get the corresponding parameters $A, B, C, m, \omega, \varphi, t_c$. With these parameters, we compute the MSE, the closeness of t_c to t_2 , denoted by $|t_c - t_2|/|t_2 - t_1|$, the damping indicator $|Bm|/|C\omega|$, and the number of oscillations $\omega \log(|t_2 - t_c|/|t_1 - t_c|)/2\pi$. Only if all criteria of a “good” fit are met, that is, $MSE < err$, $|t_c - t_2|/|t_2 - t_1| < t_{cd}$, $|Bm|/|C\omega| > d$, $0 < m < 1$, and $\omega \log(|t_2 - t_c|/|t_1 - t_c|)/2\pi > osc$, then the fit will be qualified.
3. We apply step 2 to all n time windows $[t_2 - 30, t_2], [t_2 - 35, t_2], \dots, [t_1, t_2]$, where $n = [(t_2 - t_1 - 30)/5]$, and obtain the number of qualified fits m .
4. The LPPLS Confidence Indicator is m/n .

Appendix B

Table 1. Statistics of the LPPLS Alarm in the Chinese market.

Chinese LPPLS Alarm Signal (Thresholds) — Event Study					
Positive Alarm			Negative Alarm		
Thresholds	Total Events	Valid Events	Thresholds	Total Events	Valid Events
Confidence =1	31	30	Confidence =-1	3	3
Confidence >0.9	62	60	Confidence <-0.9	3	3
Confidence >0.8	79	77	Confidence <-0.8	9	9
Confidence >0.7	100	98	Confidence <-0.7	13	13
Confidence >0.6	138	136	Confidence <-0.6	23	23
Confidence >0.5	194	191	Confidence <-0.5	29	29
Confidence >0.4	292	289	Confidence <-0.4	44	44
Confidence >0.3	393	385	Confidence <-0.3	80	80
Confidence >0.2	576	566	Confidence <-0.2	154	154
Confidence >0.1	916	894	Confidence <-0.1	330	330
Confidence >0.0	1131	1108	Confidence < 0	383	383

Chinese LPPLS Alarm Signal (Interval) — Event Study					
Positive Alarm			Negative Alarm		
Intervals	Total Events	Valid Events	Intervals	Total Events	Valid Events
Interval (1, 0.9)	92	90	Interval (-1, -0.9)	3	3
Interval (0.9, 0.8)	113	112	Interval (-0.9, -0.8)	6	6
Interval (0.8, 0.7)	87	87	Interval (-0.8, -0.7)	4	4
Interval (0.7, 0.6)	103	103	Interval (-0.7, -0.6)	10	10
Interval (0.6, 0.5)	121	120	Interval (-0.6, -0.5)	6	6
Interval (0.5, 0.4)	176	172	Interval (-0.5, -0.4)	15	15
Interval (0.4, 0.3)	241	240	Interval (-0.4, -0.3)	36	36
Interval (0.3, 0.2)	332	330	Interval (-0.3, -0.2)	74	74
Interval (0.2, 0.1)	458	445	Interval (-0.2, -0.1)	176	176
Interval (0.1, 0)	317	311	Interval (-0.1, 0)	53	53

Table 2. Statistics of the LPPLS Alarm Events in the US market.

US LPPLS Alarm Signal (Thresholds) — Event Study					
Positive Alarm			Negative Alarm		
Thresholds	Total Events	Valid Events	Thresholds	Total Events	Valid Events
Confidence =1	32	30	Confidence =-1	30	30
Confidence >0.9	76	74	Confidence <-0.9	36	36
Confidence >0.8	114	112	Confidence <-0.8	51	51
Confidence >0.7	137	134	Confidence <-0.7	60	60
Confidence >0.6	185	180	Confidence <-0.6	77	77
Confidence >0.5	237	231	Confidence <-0.5	88	88
Confidence >0.4	310	304	Confidence <-0.4	106	106
Confidence >0.3	372	366	Confidence <-0.3	134	134
Confidence >0.2	461	456	Confidence <-0.2	153	153
Confidence >0.1	525	520	Confidence <-0.1	167	167
Confidence > 0	546	541	Confidence < 0	173	173

US LPPLS Alarm Signal (Interval) — Event Study					
Positive Alarm			Negative Alarm		
Intervals	Total Events	Valid Events	Intervals	Total Events	Valid Events
Interval (1, 0.9)	76	74	Interval (-1, -0.9)	36	36
Interval (0.9, 0.8)	38	38	Interval (-0.9, -0.8)	18	18
Interval (0.8, 0.7)	23	22	Interval (-0.8, -0.7)	14	14
Interval (0.7, 0.6)	48	46	Interval (-0.7, -0.6)	22	22
Interval (0.6, 0.5)	52	51	Interval (-0.6, -0.5)	18	18
Interval (0.5, 0.4)	73	73	Interval (-0.5, -0.4)	25	25
Interval (0.4, 0.3)	62	62	Interval (-0.4, -0.3)	34	34
Interval (0.3, 0.2)	90	90	Interval (-0.3, -0.2)	23	23
Interval (0.2, 0.1)	64	64	Interval (-0.2, -0.1)	15	15
Interval (0.1, 0)	21	21	Interval (-0.1, 0)	6	6

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